Cooperative games solutions for telecommunication problems

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Game Theory and Telecommunications
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OUTLINE

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2. Basic Concepts of Bankruptcy Problems
3. Cooperative approaches to Power Control and Transmission Mode
4. Cooperative approaches to Channel Allocation
5. Game Theory and the business of sponsored search advertisements
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1. Game Theory and Telecommunication problems
   1.1. Introduction and Motivation
   1.2. Some Problems

Introduction and Motivation

1. The growth of the telecommunication industry, particularly mobile communications, has exceeded all the expectations during the last decade.

2. Nowadays there remain very few people who have not a smart device, which supposes important source of income for the companies that offer this service.

3. The mobile communications allow the communication distantly without the need of a physical link, which allows a cost reduction of installation and maintenance.

4. Cost does not depend on the distance.

5. But there are some disadvantages. This technology faces not controllable factors, as the conditions of the weather and, what is more important, the fact that the radio-electrical spectrum, on which the communications are based, is finite. The range of the spectrum is limited.
**Introduction and Motivation**

**CONSEQUENCES**

1. It is necessary to carefully **manage the available bandwidth** to guarantee an adequate level of **quality of service (QoS)** to the users.

2. It is necessary to **optimize the available (scarce) resources**.

**HOW TO DO IT?**

There are **three crucial elements** to take into account to provide an **optimal performance** of a wireless communications system:

1) **Power control** (SINR, Throughput, transmission power)

2) **Link adaptation** (SINR, Throughput, transmission power)

3) **Channel assignment** (DCA, FCA, RCA)

**Introduction and Motivation**

Objectives of the agents involved in a wireless communication system:

**Users** want to obtain as much as possible throughput with a high QoS.

The BS has **limited resources** to satisfy the users’ demands and its objective is to be able to give service as many users as possible with a reasonable QoS.
Introduction and Motivation

HOW CAN GAME THEORY HELP?

1. Game Theory deals with situations of conflict of interests, and in this kind of systems such conflictive situations arise. Users compete for scarce resources while the system tries to optimize its performance.

2. Using game theory the management of the resources of the system can be improved, taking into account the radio-electrical spectrum is limited.

3. It is possible to look for solutions to obtain a reasonable total throughput while the QoS is good enough.

4. The usual approach is from non-cooperative game theory, but also approaches from cooperative game theory could provide good results.

Some Problems

• TECHNICAL PROBLEMS:
  • Power control (SINR, Throughput, transmission power)
  • Link adaptation (SINR, Throughput, transmission power)
  • Channel assignment (DCA, FCA, RCA)

• BUSINESS PROBLEMS:
  • Business of sponsored search advertisements
  • Internet TV
2. Basic Concepts of Bankruptcy Problems

When a firm goes bankrupt, how should its liquidation value be divided among its creditors?

This is a very brief introduction to the formal analysis of problems of this kind, which we call "bankruptcy problems".

The objective of this literature, which originates in the papers by O’Neill (1982) and Aumann and Maschler (1985), is to identify well-behaved "rules" for associating with each bankruptcy problems a division between the claimants of the amount available.

Basic Concepts of Bankruptcy problems

A bankruptcy problem with set of claimants $N$ is a pair $(E, c)$ where $E$ is the estate available and $c$ is the vector of claims such that $c_i \geq 0$ for all $i \in N$ and $0 \leq E \leq \sum_{i \in N} c_i$.

Given a bankruptcy problem $(E, c)$ with set of claimants $N$, the associated (pessimistic) bankruptcy game is a TU-game $(N, v)$, where the set of players $N$ is the set of claimants, and the characteristic function $v$ is defined as follows:

$$v(S) = \max\{0, E - \sum_{i \in N \setminus S} c_i\} \text{ for all } S \subset N.$$  

These games are convex, therefore they have a nonempty core and their Shapley values are in the core.

A division rule $d$ is a function that assigns to each bankruptcy problem $(E, c)$ an allocation $d(E, c) \in \mathbb{R}^N$, such that $\sum_{i \in N} d_i(E, c) = E$ and $0 \leq d_i(E, c) \leq c_i$ for all $i \in N$.

Example:

Players:

- Jacob
- Reuben
- Simeon
- Levi
- Judah
- All
- Half
- A Third
- A fourth

Estate: $E = 120$

Claims:

- Jacob: $H = 120$
- Reuben: $120$
- Simeon: $60$
- Levi: $40$
- Judah: $30$

Game:

$$\forall S \subset N \quad v(S) = \max\{0, E - \sum_{i \in N \setminus S} c_i\} \quad \text{(convex game)}$$

$$v(R, J) = 20, \quad v(R, L) = 30, \quad v(R, S) = 50$$
$$v(R, L, J) = 60, \quad v(R, S, J) = 80,$$
$$v(R, S, L) = 90, \quad v(N) = 120,$$
$$v(T) = 0 \quad \text{otherwise}$$
Constrained Equal Award (CEA) Rule:

\[ CEA(E, c) \equiv (\min\{c_i, \lambda\})_{i \in N}, \text{ where } \lambda \text{ is chosen such that } \sum \min\{c_i, \lambda\} = E. \]

\[ = 10 \text{ u.m.} = 10 \text{ u.m.} \]
Example (cont.):

Constrained Equal Award (CEA) Rule:

\[ CEA(E, c) = (\min\{c_i, \lambda\})_{i \in N} \]

where \( \lambda \) is chosen such that

\[ \sum \min\{c_i, \lambda\} = E. \]

\[ = 10 \text{ u.m.} \]
Example (cont.):

**Constrained Equal Loss (CEL) Rule:**

\[
CEL(E, c) \equiv (\max\{c_i - \lambda, 0\})_{i \in \mathcal{N}},
\]

where \(\lambda\) is chosen such that

\[
\sum \max\{c_i - \lambda, 0\} = E.
\]

\[= 10 \text{ u.m.}\]

Reuben Simeon Levi Judah

Estate

Basic Concepts of Bankruptcy problems

02/09/2014
Example (cont.): Constrained Equal Loss (CEL) Rule:

\[ CEL(E, c) \equiv (\max \{c_i - \lambda, 0\})_{i \in N}, \]
where \( \lambda \) is chosen such that
\[ \sum_{i \in N} \max \{c_i - \lambda, 0\} = E. \]

\[ = 10 \text{ u.m.} \]

(Herrero and Martínez, 2006)
**Basic Concepts of Bankruptcy problems**

**Example (cont.):**

*Constrained Equal Loss (CEL) Rule:*

\[
CEL(E, c) \equiv \left( \max\{c_i - \lambda, 0\} \right)_{i \in N},
\]

where \( \lambda \) is chosen such that \( \sum \max\{c_i - \lambda, 0\} = E. \)

Example:

- Reuben: \( 80 \frac{10}{3} \)
- Simeon: \( 20 \frac{10}{3} \)
- Levi: \( 10/3 \)
- Judah: \( 0 \)

\( E = 10 \text{ u.m.} \)

**Other rules:**

*The Talmud (TAL) Rule:*

\[
TAL_i(E, c) = \begin{cases} 
\min\left\{ \frac{c_i}{2}, \lambda \right\} & \text{if } E \leq \sum_{j \in N} \frac{c_j}{2} \\
\frac{c_i}{2} + \min\{0, \frac{c_i}{2} - \lambda\} & \text{if } E \geq \sum_{j \in N} \frac{c_j}{2}
\end{cases}
\]

where \( \lambda \) is chosen such that \( \sum_{i \in N} TAL_i(E, c) = E. \)

*The Proportional (PROP) Rule:*

\[
PROP_i(E, c) = \frac{c_i}{\sum_{j \in N} c_j} \times E
\]
Basic Concepts of Bankruptcy problems

SOME PROPERTIES: For each \((E, c)\)

Feasibility. \(\sum R_i(E, c) \leq E\).

Efficiency. \(\sum R_i(E, c) = E\).

Non-negativity and Claim boundedness. For each \(i\), \(0 \leq R_i(E, c) \leq c_i\).

Respect of minimal rights. For each \(i\), \(R_i(E, c) \geq \max\{0, E - \sum_{j \neq i} c_j\}\).

Equal treatment of equals. For each \(i, j\) such that \(c_i = c_j\), then \(R_i(E, c) = R_j(E, c)\).

Order preservation. For each \(i, j\) such that \(c_i \geq c_j\), then \(R_i(E, c) \geq R_j(E, c)\) and \(c_i - R_i(E, c) \geq c_j - R_j(E, c)\).

Claims monotonicity. For each \(i\) and each \(c_i' > c_i\), \(R_i(E, c_i', c_i, \ldots) \geq R_i(E, c)\).

Resource monotonicity. If \(\sum c_j > E' > E\), then \(R(E', c) \geq R(E, c)\).

Etc …

3. Cooperative approaches to Power Control and Transmission Mode

3.1. Introduction

3.2. A cooperative model of power control and transmission mode
What do we mean for resource assignment in a wireless communications system?

When a user $i$ asks for service, then a Base Station (BS) essentially answers with the following parameters:

- **Channel**
- **Transmission mode**
- **Power**

**Diagram:**

- User
- Downlink
- Uplink
- Base Station

---

**Introduction**

Relationships among the involved elements in wireless communications:

$T(SINR, BLER, BER, ...) \quad SINR(power; \text{ gains, losses, noise(thermal),}...)$
A cooperative model of power control and transmission mode

Throughtput depends on the transmission mode $k$, the SINR and BLER (BLock Error Rate): 

$$T_k = \text{Throughput} = R_k (1 - \text{BLER}_k (\gamma_k))$$

where $R_k$ is the radio interface rate of mode $k$.

Then the effective throughput can be approximated by a sigmoid function of the SINR:

$$T(x) = \frac{A}{1 + e^{\lambda (x - \delta)}}$$

where $x$ is the SINR in dB, and $T(x)$ is the throughput in kbps.

Parameters for sigmoid modelling of the throughput as a function of the SINR. Source: Krishnaswamy (2002)
A cooperative model of power control (joint with link adaptation)

In view to the structure of the problem, it could also be analyzed as a bankruptcy problem. But, what would play the role of estate and claims?

A first approach: The estate and the claims are throughput.

A simple example:

\[ E = 120 \text{ Kbps}; \quad c_1 = 50 \text{ Kbps}, \quad c_2 = 40 \text{ Kbps}, \quad c_3 = 30 \text{ Kbps}, \quad c_4 = 20 \text{ Kbps} \]

If we use the CEA rule, then we obtain the following allocations:

<table>
<thead>
<tr>
<th></th>
<th>Allocation</th>
<th></th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35 Kbps (MCS-7)</td>
<td>3</td>
<td>30 Kbps (MCS-7)</td>
</tr>
<tr>
<td>2</td>
<td>35 Kbps (MCS-7)</td>
<td>4</td>
<td>20 Kbps (MCS-6)</td>
</tr>
</tbody>
</table>

Since the throughput is given in terms of the SINR, we can compute the SINR as follows:

\[
T(x) = \frac{A}{1 + e^{-\lambda x - \delta}} \rightarrow e^{\lambda x + \delta} = \frac{A - T(x)}{T(x)} \rightarrow x = \frac{\ln \left( \frac{A - T(x)}{T(x)} \right) + \lambda \cdot \delta}{\lambda} (\text{dB})
\]

- \( T_1(x) = 35 \text{kbps} \rightarrow \text{MCS-7} \rightarrow x = \frac{\ln \left( \frac{44.8 - 35}{35} \right) - 0.446}{-0.446} \rightarrow x = 17.85 \text{dB} \)
- \( T_2(x) = 35 \text{kbps} \rightarrow \text{MCS-7} \rightarrow x = \frac{\ln \left( \frac{44.8 - 35}{35} \right) - 0.446}{-0.446} \rightarrow x = 17.85 \text{dB} \)
- \( T_3(x) = 30 \text{kbps} \rightarrow \text{MCS-7} \rightarrow x = \frac{\ln \left( \frac{44.8 - 30}{30} \right) - 0.446}{-0.446} \rightarrow x = 16.58 \text{dB} \)
- \( T_4(x) = 20 \text{kbps} \rightarrow \text{MCS-6} \rightarrow x = \frac{\ln \left( \frac{29.6 - 20}{20} \right) - 0.451}{-0.451} \rightarrow x = 11.6 \text{dB} \)
A cooperative model of power control (joint with link adaptation)

Finally, the SINR depends on the level of power what allows us to compute it by means of the formula:

$$\gamma_i = SINR = \frac{PG_i}{\sum_{j \neq i} (PG_jQ_{ji}) + \eta_i}$$

Some comments:

1. The minimum SINR in code-division multiple access (CDMA) is around 6 dB. Below this level the QoS is not admissible or acceptable.

2. From this point of view, the classic bankruptcy rules which satisfies the property of *drop out* could be interesting in this context.

3. The described procedure has the following scheme:
4. Other definitions of the estate and the claims can be used. For example, the estate could be measured in terms of total SINR admissible for the system.

5. The main shortcoming of this approach could again be the intensity of uplink and downlink signaling, because this reduces the available resources.

4. Cooperative approaches to Channel Allocation

4.1. Introduction

4.2. Cooperative models of channel allocation
What do we mean for resource assignment in a wireless communications system?

When a user \( i \) asks for service, then a Base Station (BS) essentially answers with the following parameters:

- **Channel**
- **Transmission mode**
- **Power**

*The recent evolution of mobile communication systems is being characterized by high user expectations and demands in terms of quality of service (QoS) provision.*

*Such demands require the design and implementation of the necessary means to accomplish an efficient use of the scarce available resources.*

*One way to achieve such objective is through the development of Radio Resource Management (RRM) techniques.*

*An important RRM technique is channel assignment or allocation.* Channel assignment schemes are in charge of allocating, managing and distributing the available channels among users and services according to some QoS or system constraints.*
Radio resource management, and channel allocation in particular, are clear examples of potential application fields for game theory since their main functionality is to manage scarce radio resources among a set of competing users in a scenario where the actions of a particular user might affect the reminder users.

**CHANNEL ALLOCATION ISSUES:**

- Limited number of channels
- Interferencies
- Type of antennas
  - sectorized
  - omnidirectional
- Cells
- 1st and 2nd tiers of interfering cells
- Type of services (www, video, email, calls, ...)
- Homogeneous use of the channels in each cell.
Cooperative models of channel allocation

SITUATION:

A Base Station (BS) has got $c$ channels available and several users of the system ask for service to it. How should the BS distribute the available channels among them?

Possible strategies:

1. Egalitarian rule
2. Rule applying priorities of service
3. Maximizing the total throughput
4. Maximizing the minimal throughput obtained by a user.
5. Applying bankruptcy rules (scarce resource management)
6. …

Particular characteristics of the problem to take into account to apply bankruptcy rules:

1. The estate is not perfectly divisible (discrete problem)
2. The users are not identical and their happiness level (utility, in general non-linear) will depend on the type of requested service (www, mail, video, …) and the assigned resources (number of channels).
3. There are services more demanding than others
4. The problem is dynamic
Cooperative models of channel allocation

Elements of the game:

1. Estate: $R$
2. Users: $N$
3. Types of services: $S$
4. Utility functions: for each $s \in S$, $u(s): N \to [0, 1]$, where $N$ is the set of all non-negative integer numbers.

$u(s)(0) = 0$ and $u(s)(r) \leq u(s)(r + 1)$ for each $r \in N$.

If $0 < u(s)(r) < 1$, then $u(s)(r) < u(s)(r + 1)$ until saturating.

5. Priorities: Each user $i \in N$ is characterised by a pair $(s(i), t(i))$, where $s(i) \in S$ is the type of requested service and $t(i)$ is a positive integer number assigned by the system. Priority lexicographic order.

6. Claims of resources:

$r_i = \min \{ \max\{ u(s(i))(r) : r \in N \} \}

l_i = \min\{ r \in N : u(s(i))(r) > 0 \}.

Limitation in the channel allocation:

$r^{\max} < R$, $r_i = \min\{ \min \{ \max\{ u(s(i))(r) : r \in N \}, r^{\max} \} \}.

DEFINITION:

A bankruptcy situation for radio resource (channel) distribution in a communication system is defined by a 6-tuple

$M = (N, R, S, (u(s)_r)_{s \in S}, (s(i), t(i))_{i \in N}, s_i)$. where $\sum r_i > R$, otherwise the system assigns each user her claim.
Cooperative models of channel allocation

Two classical rules for bankruptcy situations:

**Constrained Equal Award (CEA) Rule:**

\[ CEA(E, c) \equiv \left( \min \{ c_i, \lambda \} \right)_{i \in N} \text{ where } \lambda \text{ is chosen such that } \sum \min \{ c_i, \lambda \} = E. \]

**Constrained Equal Loss (CEL) Rule:**

\[ CEL(E, c) \equiv \left( \max \{ c_i - \lambda, 0 \} \right)_{i \in N} \text{ where } \lambda \text{ is chosen such that } \sum \max \{ c_i - \lambda, 0 \} = E. \]

How could we adapt these two rules to the channel allocation problem?

Cooperative models of channel allocation

**DCEAM** AND **DCEL**

Step 0: Initialization.
We order the users in decreasing order of priority. For the sake of simplicity, we consider that \( 1 \leq \pi_1 \leq \pi_2 \leq \ldots \leq \pi_n \).
Consider the vector \( U^0 = (U^0, U^1, \ldots, U^n) \), where \( U^0 = u(h)(c^0) \) for all \( h \in N \).
Fix \( x^0 = 0 \) for all \( i \in N \).
Consider the set \( N^0 = \arg \min \{ U^0 : i \in N \} \) \(-\{ i \in N : x^0 = c_i \} \).

Step k: Allocation of the \( k \)-th resource.

**Step k.1: Selection of the user.**
Choose the user \( h \in N^{k-1} \) such that \( h \leq \pi_j \) for all \( j \in N^{k-1} \).
(This user is unique because we have strictly ordered all users in Step 0)

**Step k.2: Actualization of the allocation of resources**
and utilities.
Do \( x^k = x^{k-1} + 1 \) and \( U^k \Rightarrow U^{(k-1)} \) for all \( h \neq h \).
Do \( x^k = x^{k-1} + 1 \) and \( U^k = u(h)(x^k) \).

**Step k.3: Actualize the set of potential users to be served in the next step.**
Do \( N^k = \arg \max \{ U^k : i \in N \} \) \(-\{ i \in N : x^k = c_i \}. \)

**Step k.4: Step?**
If \( k = R \) then the allocation of resources among the users is \( x^R, x^{R+1}, \ldots, x^n \) and their utilities are given by vector \( UR \).
Otherwise go to Step \( k + 1 \).
Cooperative models of channel allocation

An example: $R = 15$ channels

DCEAM: $= 1$ channel

Users: $u_1, u_2, u_3, u_4, u_5$

CHANCES
Cooperative models of channel allocation

An example: $R = 15$ channels

DCEAM:

= 1 channel

0

1

Users: $u_1$, $u_2$, $u_3$, $u_4$, $u_5$

CHANNELS

47

Cooperative models of channel allocation

An example: $R = 15$ channels

DCEAM:

= 1 channel

0

1

Users: $u_1$, $u_2$, $u_3$, $u_4$, $u_5$

CHANNELS

48
Cooperative models of channel allocation

An example: $R = 15$ channels

DCEAM: $\text{CHANNELS}$

Users: $u_1$ $u_2$ $u_3$ $u_4$ $u_5$

= 1 channel
Cooperative models of channel allocation

An example: \( R = 15 \) channels

DCEAM:  
\[
\begin{array}{cccccc}
\text{Users:} & u1 & u2 & u3 & u4 & u5 \\
\text{CHANNELS} & & & & & \\
\end{array}
\]

\( = 1 \) channel

Cooperative models of channel allocation

An example: \( R = 15 \) channels

DCEAM:  
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\( = 1 \) channel
Cooperative models of channel allocation

An example: $R = 15$ channels

DCEAM:

$\text{CHANNELS}$

Users: $u_1$ $u_2$ $u_3$ $u_4$ $u_5$
Cooperative models of channel allocation

An example: $R = 15$ channels

DCEAM: = 1 channel

CHANNELS

Users: u1 u2 u3 u4 u5

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CHANNELS

Users: u1 u2 u3 u4 u5
An example: $R = 15$ channels

DCEAM: $= 1$ channel

Cooperative models of channel allocation

Users: u1, u2, u3, u4, u5

Channels
Cooperative models of channel allocation

**An example: R = 15 channels**

DCEAM: 1 channel

Users: u1, u2, u3, u4, u5

Utility: 0.6, 0.6, 0.6, 0.53, 0.4

Allocation: 2, 2, 3, 4, 4
Cooperative models of channel allocation

An example: $R = 15$ channels

DCELM: $= 1$ channel

Utility:

<table>
<thead>
<tr>
<th>Users:</th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
<th>u4</th>
<th>u5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Allocation: 3 3 5 7 10 28

Cooperative models of channel allocation

An example: $R = 15$ channels

DCELM: $= 1$ channel

Utility:

<table>
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<td>1</td>
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Allocation: 3 3 5 7 9 27
Cooperative models of channel allocation

An example: \( R = 15 \) channels

**DCELM:**

\[ = 1 \text{ channel} \]

Utility:

<table>
<thead>
<tr>
<th>Users:</th>
<th>( u_1 )</th>
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<th>( u_4 )</th>
<th>( u_5 )</th>
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</thead>
<tbody>
<tr>
<td>Allocation:</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

| CHANNELS | 26 |

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**Utility:**

<table>
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<tr>
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<td>3</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>9</td>
</tr>
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</table>

| CHANNELS | 25 |

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Cooperative models of channel allocation

An example: \( R = 15 \) channels

DCELM: \( = 1 \) channel

Utility:

Users: \( u_1, u_2, u_3, u_4, u_5 \)

Allocation: 2 2 4 7 9

Channels

Users: \( u_1, u_2, u_3, u_4, u_5 \)

Allocation: 2 2 4 6 9

Channels
Cooperative models of channel allocation

An example: $R = 15$ channels

DCELM: = 1 channel

Utility:

0 0 1 1

Users: u1 u2 u3 u4 u5

Allocation: 2 2 4 6 8 22

CHANNELS

Cooperative models of channel allocation

An example: $R = 15$ channels

DCELM: = 1 channel

Utility:

0 1

Users: u1 u2 u3 u4 u5

Allocation: 2 2 4 6 7 21
Cooperative models of channel allocation

An example: \( R = 15 \) channels

\[
\begin{align*}
\text{DCELM:} & \quad \text{Utility:} \quad \text{Allocation:} \\
& \quad = 1 \text{ channel} \\
\end{align*}
\]

Users: \( u_1 \) \( u_2 \) \( u_3 \) \( u_4 \) \( u_5 \)

Channels: | 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ |
DCELM: | 1 \ 1 \ 2 \ 2 \ 4 \ 4 \ 5 \ 5 \ 7 \ 7 \ |
Cooperative models of channel allocation

An example: $R = 15$ channels

DCELM: $\text{utility} = 1$ channel

Utility:

<table>
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<tr>
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</tr>
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<tbody>
<tr>
<td>Users</td>
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<td>u2</td>
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<tr>
<td>Allocation</td>
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</tr>
<tr>
<td>CHANNELS</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

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Cooperative models of channel allocation

An example: $R = 15$ channels

DCELM: $\text{utility} = 1$ channel

Utility:

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<td>2</td>
<td>2</td>
</tr>
<tr>
<td>CHANNELS</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>
Cooperative models of channel allocation

An example: \( R = 15 \) channels

DCELM: = 1 channel

Utility:

<table>
<thead>
<tr>
<th>Users:</th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
<th>u4</th>
<th>u5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocation:</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Channels: 16

Cooperative models of channel allocation

An example: \( R = 15 \) channels

DCELM: = 1 channel

Utility:

<table>
<thead>
<tr>
<th>Users:</th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
<th>u4</th>
<th>u5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocation:</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Channels: 15

Utility:

| | 0.6 | 0.6 | 0.4 | 0.53 | 0.5 |

Cooperative models of channel allocation

An example: \( R = 15 \) channels

DCELM: = 1 channel

Utility:

<table>
<thead>
<tr>
<th>Users:</th>
<th>u1</th>
<th>u2</th>
<th>u3</th>
<th>u4</th>
<th>u5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocation:</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Channels: 15

Utility:

| | 0.6 | 0.6 | 0.4 | 0.53 | 0.5 |
Cooperative models of channel allocation

Dynamic situation. Simulation conditions:

This simulation is based on the EDGE (Enhanced Data Rate for GSM Evolution) radio interface which allows to assign multiple channels (in this case time slots) to a single user. As a result, the radio resources to be assigned correspond to time slots (TS), and each user can be assigned a maximum of eight time slots.

The performance of the proposed multi-channel assignment policies has been assessed by means of SPHERE (Simulation Platform for HEterogeneous wiREless systems), an advanced heterogeneous system level simulation platform (López-Benitez et al., 2006). The platform integrates three advanced system level simulators emulating the GPRS, EDGE, and HSDPA radio technologies.

Utility functions:

![Utility values per traffic service and number of assigned TS.](Image)
Cooperative models of channel allocation

Dynamic situation. Simulation conditions (cont.):

- Based on the previously defined utility values, the static mechanisms assign the number of radio resources.

- If the number of radio resources to guarantee a min QoS is not available, a user served by the static assignment mechanisms is queued.

- For all channel distribution policies, all channels are redistributed each time a user requests access to the system or a user ends its transmission.

- Users with active real-time video transmissions will maintain their assigned radio resources, unless they previously received more resources than needed to guarantee their min QoS levels (in this case, only the radio resources corresponding to the min QoS level are guaranteed).

---

Some Simulation results:

**TABLE VII. DCEAM PERFORMANCE**

<table>
<thead>
<tr>
<th></th>
<th>Mean throughput (kbps)</th>
<th>Mean waiting time (seconds)</th>
<th>n° of assigned slots</th>
<th>% of aborted frames</th>
<th>% of unused frames</th>
<th>% of served users</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>54.15</td>
<td>5.3321</td>
<td>1.86</td>
<td>-</td>
<td>-</td>
<td>38.12</td>
</tr>
<tr>
<td>Email</td>
<td>34.88</td>
<td>91.8317</td>
<td>1.16</td>
<td>-</td>
<td>-</td>
<td>6.79</td>
</tr>
<tr>
<td>16kbps video</td>
<td>25.18</td>
<td>0.0171</td>
<td>1.08</td>
<td>10.51</td>
<td>3.07</td>
<td>86.53</td>
</tr>
<tr>
<td>32kbps video</td>
<td>46.53</td>
<td>0.0137</td>
<td>2.02</td>
<td>11.51</td>
<td>1.44</td>
<td>90.90</td>
</tr>
<tr>
<td>64kbps video</td>
<td>82.24</td>
<td>0.0085</td>
<td>3.71</td>
<td>13.08</td>
<td>0.12</td>
<td>97.36</td>
</tr>
</tbody>
</table>

* The mean transmission time corresponds to the time elapsed between the resource request and the end of transmission. The mean waiting time accounts for the time the user waits for resources to be assigned. The implemented real-time video model aborts a video frame if its transmission is not ended before the next video frame is to be transmitted. For long waiting times, it can also occur that a video user never gets resources for a video frame to be transmitted. The percentage of served users reflects the ratio of users that are assigned radio resources with respect to the total number of users requesting channels.
Cooperative models of channel allocation

Some Simulation results (cont.):

| TABLE VI |
| DCELM PERFORMANCE |

<table>
<thead>
<tr>
<th></th>
<th>Mean through put (kbps)</th>
<th>Mean waiting time (s)</th>
<th>n° of assigned slots</th>
<th>% of aborted frames</th>
<th>% of unsent frames</th>
<th>% of served users</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>37.31</td>
<td>2.3355</td>
<td>1.34</td>
<td>-</td>
<td>-</td>
<td>67.50</td>
</tr>
<tr>
<td>Email</td>
<td>34.10</td>
<td>84.8155</td>
<td>1.11</td>
<td>-</td>
<td>-</td>
<td>7.76</td>
</tr>
<tr>
<td>16kbps video</td>
<td>24.87</td>
<td>0.0529</td>
<td>1.04</td>
<td>13.60</td>
<td>22.69</td>
<td>56.55</td>
</tr>
<tr>
<td>32kbps video</td>
<td>36.07</td>
<td>0.0075</td>
<td>1.62</td>
<td>19.88</td>
<td>0.49</td>
<td>96.91</td>
</tr>
<tr>
<td>64kbps video</td>
<td>73.93</td>
<td>0.0065</td>
<td>3.31</td>
<td>16.33</td>
<td>0.02</td>
<td>99.68</td>
</tr>
</tbody>
</table>

Cooperative models of channel allocation

Some Simulation results (cont.):

DCEAM and DCELM user satisfaction (%)

<table>
<thead>
<tr>
<th></th>
<th>DCEAM</th>
<th>DCELM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web</td>
<td>53.02 (60.31)</td>
<td>57.48 (53.20)</td>
</tr>
<tr>
<td>Email</td>
<td>13.42 (44.27)</td>
<td>15.09 (14.90)</td>
</tr>
<tr>
<td>16kbps video</td>
<td>86.42 (70.91)</td>
<td>63.71 (86.25)</td>
</tr>
<tr>
<td>32kbps video</td>
<td>87.43 (71.08)</td>
<td>79.64 (87.41)</td>
</tr>
<tr>
<td>64kbps video</td>
<td>86.81 (70.45)</td>
<td>83.95 (87.23)</td>
</tr>
</tbody>
</table>
Cooperative models of channel allocation

Other situations in which bankruptcy problems could be useful are in heterogeneous wireless networks (4G). For these situations Lucas-Estañ et al. (2008, 2010c) and Gozalvez et al. (2008) propose a scheme, using an approach analogous to the previous one but adapted to the complexity of such networks, in order to manage situations of channel allocation with heterogeneous wireless networks (networks with several technologies). This approach is also user-oriented as in the previous model. In this user-oriented scheme the players are the users (therefore, the problem is once again dynamic) and the estate comes from the total capacity of the network technologies but taking into account different resources from different technologies cannot be combined in order to satisfy the demand of a particular user.

However, in Niyato & Hossain (2006) a simpler and network-oriented scheme for heterogeneous wireless networks based on bankruptcy techniques is proposed. In the network-oriented scheme the players are the different networks and the estate is determined by the requests of the users. Therefore, the problem is static and every time an user requests for service a bankruptcy problem is solved. In this sense, an user receives different amount of resources from different technologies. Next, we present the Niyato & Hossain’s model.

Elements of the game:

1. **Estate:** $E_k$ (requests of bandwidth of an user of type $k$)

2. **Claimants:** $N = \{\text{WLAN, CDMA cellular network, WMAN}\}$

3. **Claims:**

   $$C_i = \begin{cases} 
   b_{k,i} & \text{if } b_{k,i} < B_i^\text{available} \\
   B_i^\text{available} + u \left( B_i^\text{available} - (B_i^\text{available}) \right) & \text{if } b_{k,i} \geq B_i^\text{available} 
   \end{cases}$$

where $b_{k,i}$ is the predefined offered bandwidth by network $i$ to a new user with subscription class $k$, $B_i^\text{available}$ is the available bandwidth in network $i$, $u$ is a uniform random number between zero and one, and $r$ is a control parameter which will be referred to as the bandwidth shaping parameter (i.e., $0 < r \leq 1$).
Cooperative models of channel allocation

AN EXAMPLE (Niyato & Hossein, 2006):

Three type of users $k = 1, 2, 3$. $E_1 = 200$ Kbps, $E_2 = 350$ Kbps, $E_3 = 500$ Kbps

$B_{\text{WLAN}} = 6.2$ Mbps, $B_{\text{CDMA}} = 2.0$ Mbps, $B_{\text{WMAN}} = 50.0$ Mbps
(The bandwidth available in each technology will depend on the number of users served in each case)

$b_{1,i} = 200$ Kbps, $b_{2,i} = 150$ Kbps, $b_{3,i} = 250$ Kbps, $i = \text{WLAN, CDMA, WMAN}$

Now an user of type $k$ requests for service and there are enough bandwidth in all technologies in the BS. For each $k$ we will have a different bankruptcy game:

$$v_1(\emptyset) = 0, v_1(N) = 200, v_1(\text{wlan}) = v_1(\text{cdma}) = v_1(\text{wman}) = 0$$
$$v_1(\text{wlan, cdma}) = 0, v_1(\text{wlan, wman}) = 50, v_1(\text{cdma, wman}) = 0$$
$$v_2(\emptyset) = 0, v_2(N) = 350, v_2(\text{wlan}) = v_2(\text{cdma}) = v_2(\text{wman}) = 0$$
$$v_2(\text{wlan, cdma}) = 100, v_2(\text{wlan, wman}) = 200, v_2(\text{cdma, wman}) = 150$$
$$v_3(\emptyset) = 0, v_3(N) = 500, v_3(\text{wlan}) = 100, v_3(\text{cdma}) = 50, v_3(\text{wman}) = 150$$
$$v_3(\text{wlan, cdma}) = 250, v_3(\text{wlan, wman}) = 350, v_3(\text{cdma, wman}) = 300$$

An option to determine how much each technology contributes in satisfying the demand is to use the Shapley value (as in Niyato & Hossain (2006)). In this case is easy to compute the Shapley value because the number of agents is very small.

$$\Phi_{1,\text{wlan}} = 75, \Phi_{1,\text{cdma}} = 50, \Phi_{1,\text{wman}} = 75$$
$$\Phi_{2,\text{wlan}} = 116.6, \Phi_{2,\text{cdma}} = 91.6, \Phi_{2,\text{wman}} = 141.6$$
$$\Phi_{3,\text{wlan}} = 166.6, \Phi_{3,\text{cdma}} = 116.6, \Phi_{3,\text{wman}} = 216.6$$

(Niyato & Hossein, 2006)
5. Game Theory and the business of sponsored search advertisements

5.1. Introduction and Motivation

5.2. Cooperative model

Introduction and Motivation

• Internet has become the usual place for consumers to search for firms offering specific services.

TWO DRAWBACKS:

• Search can produce irrelevant results for the users.

• Search can produce unstructured company listings.
Introduction and Motivation

A SOLUTION:

A NEW AUCTION MARKET:

Lim & Tang (2006) used the following example:

“The GoTo search service offers firms two major benefits over the electronic Yellow Pages and other Internet search engines. First, since the firm pays GoTo the amount of its bid only when a consumer clicks on the firm’s listing, GoTo offers pay-for-performance service (i.e., variable cost of advertising instead of fixed cost). Second, since the search results appear in descending order of the bid and since the bids are revealed to all firms, each firm can submit a new bid anytime so as to change the order at which it appears on the list.”

to motivate the analysis of this new auction market.

Two real examples are the following:
Introduction and Motivation

According to this we can consider the following:

A **ranking auction market** describes a situation in which a provider offers a service for ranking several firms (the bidders) in a search tool over the Internet, for example. This process enables the bidders to obtain more clicks on the name of their firms and in this way increase their incomes. **Each bidder is interested in reaching a position which is as high as possible in the ranking**, because the number of potential customers achieved depends on its status. The provider has not established an upper bound in the number of firms to be ranked. This means that all the bidders will be included in the ranking. Thus, we are considering a market situation with **one seller** (the provider) who owns as many different objects (the positions in the ranking) as the number of buyers (the bidders) who are interested in them.

Two possible approaches from Game Theory can be considered to tackle these situations. The first approach involves analyzing the problem from a competitive point of view (Lim & Tang, 2006). The second approach is to study these situations from a cooperative perspective, in which it is interesting to examine collusive behaviour (Aparicio et al. 2009).
Cooperative model

To begin with,
A cooperative game \((N, v)\) is a **total big boss game** if the following conditions hold:

(i) \(v(S) \geq 0\), for all \(S \subset N\).
(ii) A player \(i_0\) (the big boss) such that \(v(S) = 0\), for all \(S \subset N \setminus \{i_0\}\), exists.
(iii) If \(i \in N \setminus \{i_0\}\), then \(v(S \cup i) - v(S) \leq v(T \cup i) - v(T)\), for all \(S \subset T \subset N \setminus \{i\}\).

For these games the core has the following nice structure:

\[
C(v) = \left\{ x \in \mathbb{R}^{n+1} / 0 \leq x_i \leq M_i(v), i = 1, 2, \ldots, n; x_0 = v(N) - \sum_{i=1}^{n} x_i \right\}
\]

\[M_i(v) = v(N) - v(\setminus \{i\}), i = 1, 2, \ldots, n\]

Two outstanding points in the core are the union point (UP) and the big boss point (BB):

\[
UP(v) = v(N) - \sum_{i=1}^{n} M_i(v) ; M_1(v), M_2(v), \ldots, M_n(v)
\]

\[
BB(v) = (v(N) ; 0, 0, \ldots, 0)
\]

Cooperative model

Given an ordering \(\sigma\) of the players in \(N\), the **lexicographic maximum** of the core \(C(v)\) with respect to \(\sigma\) is denoted by \(S^\sigma(v)\) and is such that:

\[
S^\sigma(v)_{\sigma(1)} = \max\{x_{\sigma(1)} : x \in C(v)\};
S^\sigma(v)_{\sigma(2)} = \max\{x_{\sigma(2)} : x \in C(v) \text{ with } x_{\sigma(1)} = S^\sigma(v)_{\sigma(1)}\};
\]

\[
\vdots
\]

\[
S^\sigma(v)_{\sigma(n)} = \max\{x_{\sigma(n)} : x \in C(v) \text{ with } (x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n-1)}) = (S^\sigma(v)_{\sigma(1)}, S^\sigma(v)_{\sigma(2)}, \ldots, S^\sigma(v)_{\sigma(n-1)})\}.
\]

The **Average Lexicographic value** (Tijs, 2005), \(Alexia\), is the average over all lexicographical maximum of the core, i.e.

\[
AL(v) = \frac{1}{n!} \sum_{\sigma \in \Theta} S^\sigma(v)
\]
In a ranking auction market (RAM) situation, the set of players can be divided into two disjoint sets \( \{0\} \) and \( B = \{1, 2, \ldots, n\} \). 0 is the provider and \( B \) are the bidders.

The provider offers a service, listing the companies according to a ranking. Each bidder \( i \in B \) bids to get the best possible position in the ranking.

The number of clients obtained depends on the position in the ranking. Assume that \( f_1 \geq f_2 \geq \ldots \geq f_n \geq 0 \), where \( f_j \) is the number of customers obtained in position \( j \).

The unitary reward per customer of bidder \( i \) is denoted by \( r_i \) and we assume, w.l.o.g., \( r_1 \geq r_2 \geq \ldots \geq r_n > 0 \).

The reward of \( i \) will be \((r_i - b_i)f_{\sigma(i)}\), where \( \sigma(i) \) denotes its position in the ranking.

The reward for the provider is given by \( \sum_{i \in B} b_if_{\sigma(i)} \).

A RAM situation can be represented by \( <0, B, r, f> \).

---

**Cooperative model**

Let \( <0, B, r, f> \) be a RAM situation. Given a subset \( S \) of \( B \), we define the RAM situation associated with \( S \) as \( \text{RAM}_S = <0, S, r_S, f_S> \), where \( r_S = (r_i)_{i \in S} \) and \( f_S = (f_j)_{1 \leq j \leq s} \) with \( s \) the cardinal of \( S \).

Let \( <0, B, r, f> \) be a RAM situation. The associated **RAM game** \((N, v)\) is a cooperative game with set of players \( N = \{0\} \cup B \) and characteristic function

\[
\nu(0) = 0 \text{ and } \nu(S) = 0, \text{ for all } S \subset B;
\]

\[
\nu(S \cup 0) = \max \{\sum_{i \in S} r_if_{\sigma(i)}\}, \text{ otherwise.}
\]

**EXAMPLE:** Consider a RAM situation with the provider and 3 bidders in which the reward vector is \((8, 3, 2)\) and the number of clients obtained according to the ranking is, respectively, \((5, 4, 2)\). The corresponding RAM game \((\{0, 1, 2, 3\}, \nu)\) is

\[
\nu(01) = 40; \nu(02) = 15; \nu(03) = 10; \nu(012) = 52; \nu(013) = 48; \nu(023) = 23; \nu(0123) = 56 \text{ and } \nu(S) = 0, \text{ otherwise.}
\]
THEOREM:
Let \( <0,B,r,f> \) be a RAM situation. The associated RAM game \((N,v)\) is a total big boss game.

Since a RAM game is a big boss game, then it is easy to determine its core:

\[
C(v) = \left\{ x \in \mathbb{R}^{n+1} \mid 0 \leq x_i \leq M_i(v), i = 1,2,...,n; x_0 = \sum_{i=1}^{n} r_i f_i - \sum_{i=1}^{n} x_i \right\}
\]

\[
M_i(v) = r_i f_i - \sum_{j=i+1}^{n} r_j (f_j - f_i), i = 1,2,...,n-1
\]

\[
M_n(v) = r_n f_n
\]

In the previous section, based on the paper by Lim & Tang (2006), several conclusions regarding the players behaviour have been presented.

This behaviour is related to how relevant the position is in the ranking for the profits achieved.

Lim & Tang (2006) also give some indications of which circumstances in the ranking are in favour of the provider and which are beneficial to the bidders.

One of the main ideas is the rank-sensitiveness. For two players it is clear but for more than two players it is necessary a precise definition.

DEFINITION \( F^k = \{ f \in \mathbb{R}^n : f_1 \geq f_2 \geq ... \geq f_n \geq 0 \text{ and } \sum f_i = k \} \)

Given \( f, g \in F^k \), it is said that \( f \) is more rank-sensitive than \( g \), or \( f \) rank dominates \( g \), if \( f \geq_R g \). If

\[
\sum_{i=1}^{k} f_i \geq \sum_{i=1}^{k} g_i, \text{ for all } k = 1,2,...,n.
\]
For RAM games Alexia is given by

\[ AL(v) = \left\{ v(N) - \frac{1}{2} \sum_{m=1}^{n-1} m(r_m - r_{m+1}) f_m + nr_m f_n, \frac{1}{2} \sum_{m=1}^{n-1} m(r_m - r_{m+1}) f_m + nr_m f_n \right\}, i \in B \]

**THEOREM**

Let \( < 0, B, r, f > \) and \( < 0, B, r, f' > \) be ranking auction market situations with \( n \leq 3 \) and let \((N, v)\) and \((N, v')\) be the associated ranking auction market games. If \( f' \geq_R f \), then \( AL_0(v') \geq AL_0(v) \).

**EXAMPLE**

Consider the RAM situation with \( r = (1, 1, 1, 0.0001) \),

\( f' = (80, 10, 10, 0) \) and \( f = (70, 20, 5, 5) \).

In this situation, we obtain \( AL_0(v) = 87.50025 > 85.0015 = AL_0(v') \).

As in the non cooperative model with two players, it is not enough the rank-sensitiveness to explain which is the better situation for the provider (bidders).

**PROPOSITION**

Let \( < 0, B, r, f > \) and \( < 0, B, r, f' > \) be RAM situations with \((N, v)\) and \((N, v')\) the corresponding RAM games. If

1. \( r \) is such that

\[ (r_1 - r_2) \leq 2(r_2 - r_3) \leq \ldots \leq (n - 1)(r_{n-1} - r_n) \leq nr_n \neq 0; \] and

2. \( f' \geq_R f \),

then \( AL_0(v) \leq AL_0(v') \) and \( \Sigma_{i \in B} AL_i(v) \geq \Sigma_{i \in B} AL_i(v') \).
EXAMPLE Consider the following situations

\[ \text{RAM} = < 0, \{1, 2\}, (10, 2), (15, 5) > \]
\[ \text{RAM}' = < 0, \{1, 2\}, (10, 2), (20; 0) > \]

It holds that \((r_1 - r_2) = 8 > 2r_2 = 4\). It is easy to check that

\[ AL_0(v) = 90 < 120 = AL_0(v') \]
\[ AL_1(v) + AL_2(v) = 70 < 80 = AL_1(v') + AL_2(v'). \]

PROPOSITION

Let \(< 0, B, r, f >\) and \(< 0, B, r, f' >\) be RAM situations with \((N, v)\) and \((N, v')\) the corresponding RAM games. If

1. \(r\) is such that

\[(r_1 - r_2) \leq (r_2 - r_3) \leq \ldots \leq (r_{n-1} - r_n) \leq r_n \neq 0; \text{ and} \]

2. \(f' \succeq_R f, \)

then \(AL_0(v) \leq AL_0(v')\) and \(AL_i(v) \geq AL_i(v')\) for all \(i \in B.\)
EXAMPLE Consider the following situations

\[
\begin{align*}
\text{RAM} &= <0, \{1, 2\}, (10, 3), (15, 5)> \\
\text{RAM}' &= <0, \{1, 2\}, (10, 3), (16, 4)>
\end{align*}
\]

In this case, we have \((r_1 - r_2) = 7 > r_2 = 3\) and, then,

\[
\begin{align*}
AL_0(v) &= 97.5 < AL_0(v') = 104 \\
AL_i(v) &= 60, \ AL_i(v') = 7.5, \ AL_1(v') = 62, \ AL_2(v') = 6.
\end{align*}
\]

COROLLARY

Let \(<0, B, r, f'>\) and \(<0, B, r, f'>\) be RAM situations with \((N, v)\) and \((N, v')\) the corresponding RAM games. If

1. \(r\) is such that \(r_i = r_j\) for all \(i, j \in B\) and
2. \(f' \geq_R f\),

then \(AL_i(v) \leq AL_i(v')\) and \(AL_i(v') \geq AL_i(v)\) for all \(i \in B\).

THEOREM

Let \(<0, B, r, f'>\) and \(<0, B, r, f'>\) be RAM situations with \((N, v)\) and \((N, v')\) the corresponding RAM games. If

1. \(r\) is such that \(r_i = r_j\) for all \(i, j \in B\) and
2. \(f' \geq_R f\),

then \(\Phi_i(v) \leq \Phi_i(v')\) and \(\Phi_i(v') \geq \Phi_i(v)\) for all \(i \in B\). \((\Phi \text{ is the Shapley value})\)
Finally, we study how the **collusion game** is.

Let \(< 0, B, r, f>\) be a RAM situation and let \((N, v)\) be the associated RAM game. The dual game restricted to all players except the big boss, the **bidders (collusion) game**, \((B, v^*)\) is given, for each \(S \subset B\), by

\[
v^*(S) = v(N) - v(N \setminus S) = \sum_{k=1}^{n-s} (r_k - r_i)f_k + \sum_{k=n-s+1}^{n} r_k f_k
\]

where \(N \setminus S = \{i_1, i_2, \ldots, i_{n-s}\}\) such that \(i_1 < i_2 < \ldots < i_{n-s}\).

In a certain sense, this represents the idea that the bidders in \(S\) think that the bidders outside \(S\) will bid as much as possible, so they have to bid in order to compensate for this. Therefore, this is a pessimistic approach.

If we consider the symmetric case, i.e. \(r_i = r\) for all \(i \in B\), then the dual game is given, for each \(S \subset B\), by

\[
v^*(S) = v(N) - v(N \setminus S) = r \sum_{k=n-s+1}^{n} f_k
\]

Moreover, if the position in the ranking is not relevant, i.e., \(f_j = f\), for all \(j = 1, 2, \ldots, n\), then the dual game is

\[
v^*(S) = v(N) - v(N \setminus S) = f \sum_{i \in S} r_i
\]

for each \(S \subset B\).

In these two situations is easy to compute the Shapley value of the game or other solutions.
6. Game Theory and the business of Internet TV

6.1. Introduction and Motivation

6.2. Cooperative iTV games

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**Introduction and Motivation**

**Internet television** (Internet TV, Online TV) is a general term that covers the delivery of television shows and other video content on the Internet by video streaming technology. It does not describe a technology used to deliver content.

**Web television** is a similar term often used to characterize short programs or videos created by a wide variety of companies and individuals.

Internet television allows the users to choose the content or the television show they want to watch from an archive of content or from a channel directory. The two forms of viewing Internet television are streaming the content directly to a media player or simply downloading the media to the user's computer.

An archive is a collection of information and media much like a library or interactive-storage facility. However, these archives can vary from a few weeks to months to years, depending on the archivist and the type of content.
Introduction and Motivation

The benefit of large archives, is that they bring in far more users who, in turn, watch more media, leading to a wider audience base and more advertising revenue. Large archives will also mean the user will spend more time on that website rather than a competitors, leading to starvation of demand for the competitors.

However, large archives are expensive to maintain, server farms and mass storage is needed along with ample bandwidth to transmit it all. Vast archives can be hard to catalogue and sort so that it is accessible to users.

Therefore, with the "TV on Demand" market growing, many providers of Internet television services exist including conventional television stations that have taken advantage of the Internet as a way to continue showing television shows after they have been broadcast often advertised as "on-demand" and "catch-up" services.

Introduction and Motivation

Internet Protocol television (IPTV) is a system through which television services are delivered using the Internet protocol suite over a packet-switched network such as the Internet, instead of being delivered through traditional terrestrial, satellite signal, and cable television formats.

IPTV services may be classified into three main groups:

• **Live television**, with or without interactivity related to the current TV show;
• **Time-shifted television; catch-up TV** (replays a TV show that was broadcast hours or days ago), **start-over TV** (replays the current TV show from its beginning);
• **Video on demand (VOD)**: browse a catalog of videos, not related to TV programming.

IPTV is distinguished from Internet television by its on-going standardization process (e.g., European Telecommunications Standards Institute) and preferential deployment scenarios in subscriber-based telecommunications networks with high-speed access channels into end-user premises via set-top boxes or other customer-premises equipment.
**Introduction and Motivation**

Broadcasting rights, distribution rights, property rights, copy rights, etc. vary from country to country. These rights govern the distribution of copyrighted content and media and allow the sole distribution of that content at any one time.

Those rights can also be restricted to allowing a broadcaster or TV-service provider rights to distribute that content for a limited time.

For example, some companies pay very large amounts for broadcasting rights with sports.

With the exception of Internet-connectivity costs many online-television channels or sites are free. These sites maintain this free-television policy through the use of video advertising, short commercials and banner advertisements may show up before a video is played. An example of this is in place of the advertisement breaks on normal television, a short thirty-second advertisement is played.

Other important aspects to take into account in the Internet TV are:

- targeted advertisements and
- the concept “any device, anywhere, anytime”.

Introduction and Motivation

• iTV Service Providers
  Their Strategies: Fees for end-users (as many end-users as possible)
                   Fees for ads (targeted, broadcast, banners) (as many ads as possible)
                   How much to pay to Content producers? (as many contents as possible)

• Content producers
  Their Strategies: How much to get from their contents? (as much as possible)
                   How many iTV Services Providers to collaborate with?

• Advertisers
  Their Strategies: How much to pay to iTV Service Providers?
                   To obtain as many viewers as possible (likely interested on their products)

• End-users
  Their Strategies: How much to pay per view? Or How many ads to watch per view?
                   Which iTV Services Providers to choose?

Cooperative iTV games

Consider the following situation:

• There is a **single provider of Internet TV**.

• There is a set of agents (video content producers) that have video contents (we initially assume that each agent has exactly one video content).

• If an end-user of the iTV watches a video content, a benefit is obtained either from ads that are inserted into it, or from pay per view.

• Likewise we consider that an end-user enters in the iTV-system can move from one video content $i$ to other video content $j$ which is recommended or suggested by the system (we initially consider that an end-user cannot watch a video content after visitint more than one video content in the iTV system). This assumption can be easily relaxed as we will explain in the concluding and further remarks.
Cooperative iTV games

iTV SYSTEM

Cooperative iTV games

NOTATION:

• iTV provider: 0

• Set of video contents: \( C = \{1, 2, 3, \ldots, c\} \)

  For each video content \( i \) we consider two elements:
  • \( D_i \) profit obtained when an end-user watches video content \( i \).
  • \( n_i \) number of end-users first visiting video content \( i \).

• A transition matrix:

\[
T = \begin{bmatrix}
    p_{i1} & \cdots & p_{ic} \\
    \vdots & \ddots & \vdots \\
    p_{ci} & \cdots & p_{cc}
\end{bmatrix}
\]

where \( p_{ij} \) is the proportion of end-users who visit content \( i \) and go to content \( j \).

• \( p_{ij} = 0 \). (1) nobody is interested in visiting \( j \) from \( i \); (2) \( j \) is not suggested in \( i \).

• \( \sum_{i \in C} p_{ij} \leq 1 \). It is possible to leave the iTV system without watching any content.
Cooperative iTV games

An Internet TV system is completely described by an iTV provider \((0)\), a set of video contents \((C)\), a transition matrix \((T)\), a profit vector \((D = (D_1, D_2, \ldots, D_c))\) and a demand of contents vector \((n = (n_1, n_2, \ldots, n_c))\). Thus, iTV = \((0, C, T, D, n)\).

Definition of an iTV game:

Given an Internet TV system, iTV = \((0, C, T, D, n)\), a iTV game is defined as follows:

- **Set of players**: \(C \cup 0\)
- **Characteristic function**: for every coalition \(S \subset C \cup 0\)

\[
\begin{align*}
  v(\emptyset) &= 0; \\
  v(\{0\}) &= 0; \\
  v(S) &= 0, \forall S \subset C; \\
  v(S \cup 0) &= \sum_{i \in S} D_i \sum_{i \in S} p_{ij}, \geq 0, \forall S \subset C.
\end{align*}
\]

A game \((C \cup 0, v)\) is said to be a **Big Boss game** (Muto et al. 1988) if it satisfies the following three conditions:

1. \(v(S) = 0, \forall S \subset C \text{ and } v(0) = 0\);
2. \(v(T \cup 0) \geq v(S \cup 0) \geq 0, \forall S \subset T \subset C\);
3. \(v(C \cup 0) - v(C \cup 0 \setminus T) \geq \sum_{i \in T} [v(C \cup 0) - v(C \cup 0 \setminus i)], \forall T \subset C\).

Q. Are iTV games Big Boss games?

**EXAMPLE**: \(0, C = \{1, 2, 3\}, D = (3, 4, 5), n = (16, 18, 25), T = \begin{bmatrix} 5 & 1 & 4 \\ 5 & 1 & 4 \\ 5 & 1 & 4 \end{bmatrix}
\)

\(v(01) = 24, v(02) = 24, v(03) = 50, v(012) = 64, v(013) = 109, v(023) = 109, v(0123) = 184\)

\(v(0123) - v(01) = 184 - 24 = 160\)

\(v(0123) - v(013) = 184 - 109 = 75\)

\(v(0123) - v(012) = 184 - 64 = 120\)

This game is not a Big Boss game!
**Cooperative iTV games**

Q. How are marginal contributions in iTV games?

\[ v(C \cup 0) - v(C \cup 0 \setminus T) = \sum_{j \in T} \left\{ D_j \sum_{n \in T} n_j p_n \right\} + \sum_{j \in C \setminus T} \left\{ D_j \sum_{n \in T} n_j p_n \right\} + \sum_{j \in C \setminus T} \left\{ D_j \sum_{n \in C \setminus T} n_j p_n \right\} \]

Q. Are iTV games convex?

A game \((N, v)\) is **convex** if it satisfies the following condition:

\[ v(S \cup i) - v(S) \leq v(T \cup i) - v(T), \forall i \in N, \forall S \subseteq T \subseteq N \setminus i. \]

Therefore, iTV games are convex! iTV games have **nonempty core**! (Shapley, 1971)
Cooperative iTV games

Q. How is the core of an iTV game?

If for each cell $ij$ in the transition matrix $T$ we consider the following allocation:

$$D_i n_i p_i = 0_j D_j n_i p_i + o_j D_j n_i p_i + d_j D_j n_i p_i,$$

where $0_j + o_j + d_j = 1$, $0_j$, $o_j$, $d_j \geq 0$.

**THEOREM.** Given an iTV game, $(x_0; x_1, \ldots, x_c)$ belongs to the core if and only if it can be written as follows:

$$x_0 = \sum_{j \in C} \sum_{i \in C} 0_j D_i n_i p_i,$$

$$x_i = \sum_{j \in C} o_j D_i n_i p_i + \sum_{j \in C} d_j D_j n_i p_i, \forall i \in C.$$

where $0_j + o_j + d_j = 1$, $0_j$, $o_j$, $d_j \geq 0$.

**EXAMPLE:** 0, $C = \{1, 2, 3\}$, $D = (3, 4, 5)$, $n = (16, 18, 25)$, $T = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix}$

$v(01) = 24$, $v(02) = 24$, $v(03) = 50$, $v(012) = 64$, $v(013) = 109$, $v(023) = 109$, $v(0123) = 184$

$\text{Benefit} = \begin{bmatrix} 16 & 18 & 25 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$ $\begin{bmatrix} 4 \\ 0 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 24 & 16 & 20 \\ 0 & 24 & 15 \\ 15 & 20 & 50 \end{bmatrix}$

$\text{Shares} = \begin{bmatrix} (1,0,0) & (0,0,1) & (0,0,1) \\ (0,1,0) & (0,\frac{1}{2},\frac{1}{2}) & (0,1,0) \\ (0,1,0) & (0,\frac{1}{2},\frac{1}{2}) \end{bmatrix} = \begin{bmatrix} (24,0,0) & (0,0,16) & (0,0,20) \\ (0,0,0) & (0,12,12) & (0,15,0) \\ (0,15,0) & (0,0,20) & (0,25,25) \end{bmatrix}$

$(x_0; x_1, x_2, x_3) = (24,0,75,85)$
Cooperative iTV games

If we are interested in symmetric distributions of the obtained profit; and we accept that the previous procedure to distribute is intuitive and reasonable, then we would have the following:

\[ o_y = p, o_d = o, d_y = d, \forall i, j \in C. \]

That means that the provider, the origin contents and the destination contents always obtain the same share of the obtained profit, although these shares could be different each other.

Therefore there are infinite ways to allocate symmetrically and coalitionally stable the generated profit.

Note that for the diagonal cells the content gets money from two different ways, one as origin and another as destination. Somehow this fact can be considered reasonable.

Cooperative iTV games

As previously stated, iTV games are not Big Boss games but they have many similarities with other class of games, TFTS-games.

TFTS-games were introduced by van den Nouweland et al. (1996).

TFTS games arise from the sum of two games, one 1-game and one 2-game.

A cooperative game \((N, v)\) is a \(k\)-game if its characteristic function can be written as follows:

\[ v(S) = \sum_{T \subseteq \mathcal{P}^k} v(T), \forall S \subseteq N. \]

Therefore, a TFTS game is given by

\[ v(S) = \sum_{i \in S} v(i) + \sum_{i, j \in S} v(ij), \forall S \subseteq N. \]
Cooperative iTV games

Given an iTV system $(0, C, T, D, n)$ we define the following two games over the set of video contents $C$:

\[ w^1(S) = \sum_{i \in S} D_{ij} p_j, \forall S \subseteq C. \]

\[ w^2(S) = \sum_{i,j \in S} D_{ij} p_i, \forall S \subseteq C. \]

It is easy to check $(C, w^1)$ is a 1-game and $(C, w^2)$ is a 2-game. In this sense $(C, w^1)$ measures the profit when the video contents collaborate with the provider but they do not interact each other. And $(C, w^2)$ measures the positive synergies among the different video contents when they collaborate with the provider.

Therefore an iTV game $(C \cup 0, v)$ can be written as follows:

\[ v(S \cup 0) = w^1(S) + w^2(S), \forall S \subseteq C. \]

Remark: These games defined over $N \cup 0$ are a 2-game and a 3-game, respectively, defined over $N$.

Cooperative iTV games

A cooperative game $(N \cup 0, v)$ is a TFFS Big Boss game (or an iTV game) if it satisfies the following three conditions:

1. $v(S) = 0, \forall S \subseteq N$ and $v(0) = 0$;
2. $v(T \cup 0) \geq v(S \cup 0) \geq 0, \forall S \subseteq T \subseteq N$;
3. $v(S \cup 0) = w^1(S) + w^2(S), \forall S \subseteq N$.

where $w^1$ and $w^2$ are a 1-game and a 2-game, respectively, defined over $N$.

Remark: These games defined over $N \cup 0$ are a 2-game and a 3-game, respectively.

Now marginal contributions are given as follows:

\[ M_i = v(C \cup 0) - v(C \cup 0) = \sum_{j \in C} w^1(i) + \sum_{j \in C \cap S} w^2(i); \]

\[ M = w^1(i) + \sum_{j \in C \cap S} w^2(ij), \forall i \in C. \]
### Cooperative iTV games

#### THE SHAPLEY VALUE:

\[
\phi_0 = \frac{1}{2} \sum_{i \in C} w^i(i) + \frac{1}{3} \sum_{i,j \in C, i < j} w^i(j);
\]

\[
\phi_i = \frac{1}{2} w^i(i) + \frac{1}{3} \sum_{j \in C \setminus \{i\}} w^i(j), \forall i \in C.
\]

Shares = \[
\begin{pmatrix}
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) \\
\vdots & \ddots & \vdots \\
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2})
\end{pmatrix}
\]

#### THE TIJS VALUE:

\[
\tau_0 = \alpha M_0, \quad \tau_i = \alpha M_i, \forall i \in C.
\]

where

\[
M_0 = v(C \cup \emptyset) - v(C) = v(C \cup \emptyset) - \sum_{i \in C} w^i(i) + \sum_{i,j \in C, i < j} w^i(j);
\]

\[
M_i = w^i(i) + \sum_{j \in C \setminus \{i\}} w^i(j), \forall i \in C; \text{ and}
\]

\[
\alpha = \frac{v(C \cup \emptyset)}{3v(C \cup \emptyset) - \sum_{i \in C} w^i(i)}.
\]

**Remark:** For iTV games the Shapley value and the Tijs value do not coincide in general. However, there are two simple situations in which both coincide:

1. \( \sum_{i \in C} w^i(i) = v(C \cup \emptyset) \).
2. \( \sum_{i \in C} w^i(i) = 0 \).

**THEOREM:** The Tijs value of an iTV game belongs to the core.

---

#### OTHER VALUES:

#### Shares:

\[
\gamma_0 = \frac{1}{3} \sum_{i \in C} w^i(i) + \frac{1}{3} \sum_{i,j \in C, i < j} w^i(j);
\]

\[
\gamma_i = \frac{1}{2} w^i(i) + \frac{1}{3} \sum_{j \in C \setminus \{i\}} w^i(j), \forall i \in C.
\]

Shares = \[
\begin{pmatrix}
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) \\
\vdots & \ddots & \vdots \\
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2})
\end{pmatrix}
\]

\[
\delta_0 = \frac{1}{2} \sum_{i \in C} w^i(i) + \frac{1}{3} \sum_{i,j \in C, i < j} w^i(j);
\]

\[
\delta_i = \frac{1}{2} w^i(i) + \frac{1}{4} \sum_{j \in C \setminus \{i\}} w^i(j), \forall i \in C.
\]

Shares = \[
\begin{pmatrix}
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) \\
\vdots & \ddots & \vdots \\
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2})
\end{pmatrix}
\]

\[
\eta_0 = \frac{1}{4} \sum_{i \in C} w^i(i) + \frac{1}{4} \sum_{i,j \in C, i < j} w^i(j);
\]

\[
\eta_i = \frac{3}{4} w^i(i) + \frac{1}{4} \sum_{j \in C \setminus \{i\}} w^i(j) + \frac{1}{4} \sum_{j \in C \setminus \{i\}} D_n, p_n, \forall i \in C.
\]

Shares = \[
\begin{pmatrix}
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) \\
\vdots & \ddots & \vdots \\
(\frac{1}{2}; \frac{1}{2}; \frac{1}{2}) & \cdots & (\frac{1}{2}; \frac{1}{2}; \frac{1}{2})
\end{pmatrix}
\]
Cooperative iTV games

EXAMPLE: 0, \( C = \{1, 2, 3\} \), \( D = (3, 4, 5) \), \( n = (16, 18, 25) \), \( T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \)

\( v(01) = 24, v(02) = 24, v(03) = 50, v(012) = 64, v(013) = 109, v(023) = 109, v(0123) = 184 \)

\[ \begin{bmatrix} 24 & 16 & 20 \\ 15 & 20 & 50 \end{bmatrix} \]

\[ \text{Benefit} = \begin{bmatrix} (12,6,6) & (\frac{12}{25},\frac{12}{25},\frac{12}{25}) & (\frac{12}{25},\frac{12}{25},\frac{12}{25}) \\ (0,0,0) & (12,6,6) & (5,5,5) \end{bmatrix} \]

\[ \Phi = (77\frac{1}{4};29,29,48\frac{1}{3}) \]

\[ \gamma = (61\frac{1}{4};33,33,56\frac{1}{3}) \]

Cooperative iTV games

SEVERAL COMMENTS ABOUT THIS MODEL:

1. The highest the number of video contents is, the highest the number of end-users is. That means \( n(c) \leq n(c + 1) \) for all \( c \geq 0 \). (Under certain conditions on \( n(c) \) the game could be a Big Boss game).

\[ n_i(c) - n_i(c - t) \geq (t - s + 1)(n_i(c) - n_i(c - s)), \forall c \geq t \geq s \]

2. Transition matrix \( T \) can be interpreted in the following way: each cell \( p_{ij} \) represents the proportion of end-users entering through \( i \) that watch video content \( j \). Hence we're only interested in initial and the final video contents. So defined matrix \( T \) is not a transition matrix properly. However, if we consider a real transition matrix \( T \), What is the role of \( T^k \)?

3. A content producer can very often have more than one video content, in this case we have two possible situations:

1. If each video content is individually treated, then we are in the same situation.
2. If all video contents belonging to the same content producer have to be equally treated, then the situation is a little bit different from the previous one, although the results seem “essentially” the same.
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AND
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