Non-cooperative games solutions for telecommunication problems

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Game Theory and Telecommunications

Lake Como School of Advanced Studies
September 7-12, 2014

CENTRO DE INVESTIGACION OPERATIVA

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OUTLINE

1. Game Theory and Telecommunication problems

2. Non-cooperative approaches to Power Control and Transmission Mode

3. Non-cooperative approaches to Channel Allocation

4. Game Theory and the business of sponsored search advertisements

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1. Game Theory and Telecommunication problems

1.1. Introduction and Motivation

1.2. Some Problems

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Introduction and Motivation

1. The growth of the telecommunication industry, particularly mobile communications, has exceeded all the expectations during the last decade.

2. Nowadays there remain very few people who have not a smart device, which supposes important source of income for the companies that offer this service.

3. The mobile communications allow the communication distantly without the need of a physical link, which allows a cost reduction of installation and maintenance.

4. Cost does not depend on the distance.

5. But there are some disadvantages. This technology faces not controllable factors, as the conditions of the weather and, what is more important, the fact that the radio-electrical spectrum, on which the communications are based, is finite. **The range of the spectrum is limited.**

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Introduction and Motivation

CONSEQUENCES

1. It is necessary to carefully **manage the available bandwidth** to guarantee an adequate level of **quality of service** (QoS) to the users.

2. It is necessary to **optimize the available (scarce) resources**.

HOW TO DO IT?

There are **three crucial elements** to take into account to provide an **optimal performance** of a wireless communications system:

1) **Power control** (SINR, Throughput, transmission power)

2) **Link adaptation** (SINR, Throughput, transmission power)

3) **Channel assignment** (DCA, FCA, RCA)

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Introduction and Motivation

Objectives of the agents involved in a wireless communication system:

Users want to obtain as much as possible throughput with a high QoS.

The BS has limited resources to satisfy the users' demands and its objective is to be able to give service as many users as possible with a reasonable QoS.

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Introduction and Motivation

HOW CAN GAME THEORY HELP?

1. Game Theory deals with situations of conflict of interests, and in this kind of systems such conflictive situations arise. **Users compete for scarce resources** while the system tries to optimize its performance.

2. Using game theory the management of the resources of the system can be improved, taking into account the radio-electrical spectrum is limited.

3. It is possible to look for solutions to obtain a reasonable total throughput while the QoS is good enough.

4. The usual approach is from **non-cooperative game theory**, but also approaches from **cooperative game theory** could provide good results.

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Some Problems

- TECHNICAL PROBLEMS:
  - **Power control** (SINR, Throughput, transmission power)
  - **Link adaptation** (SINR, Throughput, transmission power)
  - **Channel assignment** (DCA, FCA, RCA)
- BUSINESS PROBLEMS:
  - **Business of sponsored search advertisements**
  - **Internet TV**

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2. Non-cooperative approaches to Power Control and Transmission Mode

2.1. Introduction

2.2. A non-cooperative model of power control and transmission mode

Introduction

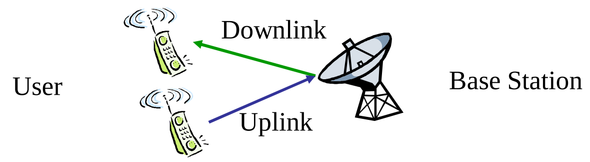
What do we mean for resource assignment in a wireless communications system?

When a user *i* asks for service, then a Base Station (BS) essentially answers with the following parameters:

Channel

Transmission mode

Power

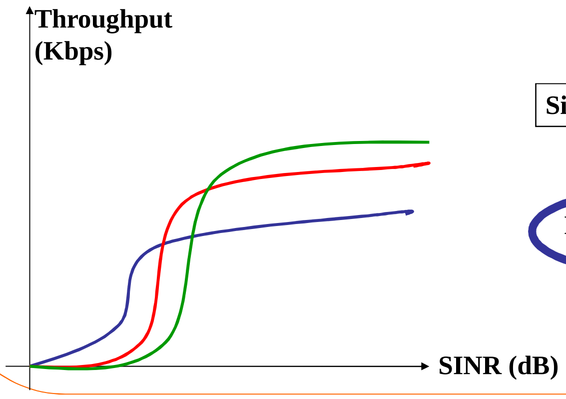


Introduction

Relationships among the involved elements in wireless communications:

SINR **THROUGHPUT** **POWER TRANSMISSION**

T(SINR, BLER, BER,..) SINR(power, gains, losses, noise(thermal),...)



Sigmoidal functions → **Link Adaptación**

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A non cooperative model of power control and transmission mode

Some references:

Goodman & Mandayam (2000) → Utility functions and trade-off between SINR and interferences (prices)

Mackenzie & Wicker (2001a) → Utility functions and trade-off between SINR and transmission power

Krishnaswamy (2002) → Utility functions and effective throughput (sigmoidal approximation), SINR and penalty functions

Other references:
Sung & Wong (2003)
Mackenzie & Wicker (2001b, 2001c)

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A non cooperative model of power control and transmission mode

Following Krishnaswamy (2002), we consider a wireless data system with the following charasteristics:

N_b **base stations**,
N_p **levels of power**,
N_m **transmission modes**,
N_u **users in the system**
and N_c **available channels**.

The allocation of resources is given by a 3-tuple (c_i, m_i, P_i), $i \in N_u$, channel, mode and power.

In this conditions the SINR received for a user is given by

$$\gamma_i = SINR = \frac{P_i G_{ii}}{\left[\sum_{j \neq i} \left(P_j G_{ji} Q_{j,ci}\right) + \eta_i\right]}$$

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A non cooperative model of power control and transmission mode

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Assigned power to i

Path loss between i and its BS

$$\gamma_i = SINR = \frac{P_i G_{ii}}{\left[\sum_{j \neq i} \left(P_j G_{ji} Q_{j,ci}\right) + \eta_i\right]}$$

Thermal noise received at i

Path loss between i and BS serving j

Occupancy of channel c_i

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A non cooperative model of power control and transmission mode

Throughtput depends on the transmission mode k, the SINR and BLER (BLock Error Rate):

$$T_k = Throughput = R_k(1 - BLER_k(\gamma_i))$$

where R_k is the radio interface rate of mode k.

Then the effective throughtput can be approximated by a sigmoid function of the SINR:

$$T(x) = \frac{A}{\left(1 + e^{-\lambda(x-\delta)}\right)},$$

where x is the SINR in dB, and $T(x)$ is the throughput in kbps.

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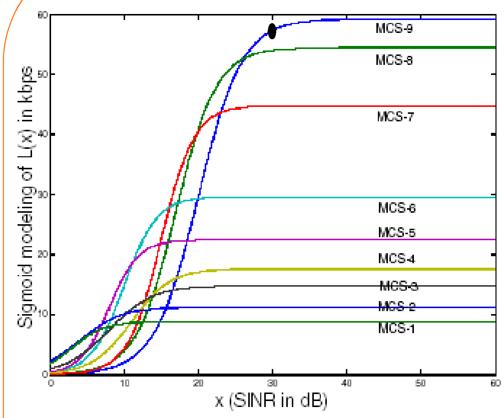
A non cooperative model of power control and transmission mode



Sigmoid modelling for all nine EGPRS MCS schemes. Source: Krishnaswamy (2002)

Curvature

$$X = \max C(x) = \left| T''(x) \middle/ \left(1 + (T'(x))^2\right)^{3/2} \right|$$

Upper knee of the sigmoid function

MCS	A	δ	λ
MCS-9A	59.2	20	0.338
MCS-8A	54.4	17	0.367
MCS-7B	44.8	15	0.446
MCS-6A	29.6	10	0.451
MCS-5B	22.4	8	0.505
MCS-4C	17.6	11	0.379
MCS-3A	14.8	8	0.337
MCS-2B	11.2	4	0.357
MCS-1C	8.8	3	0.454

Parameters for sigmoid modelling of the throughput as a function of the SINR. Source: Krishnaswamy (2002)

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A non cooperative model of power control and transmission mode

The utility function for each user is given by

$$U(x) = \begin{cases} T(x), & x \leq X_u \\ T(x) - \alpha(x - X_u), & x > X_u \end{cases}$$

where $\alpha < T'(X_u)$.

Consider now a non cooperative game with set of players BSs and users. The objective of the game is for each player to use an **optimal power level** so as to obtain **maximum throughput** while causing **minimal interference** in the network. The optimal solution obtained by setting $U'(x) = 0$ corresponds to a Nash equilibrium.

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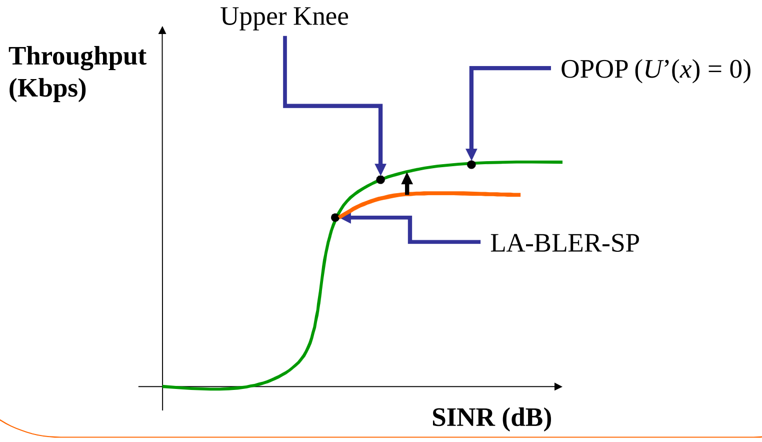
A non cooperative model of power control and transmission mode

All users need a channel for transmitting. The channel must have a suitable conditions for transmitting, and furthermore the user must **control the power level** to avoid interferences and not be punished. Therefore, this system optimizes the porwer control and at the same time the **link adaptation**, because if the channel does not have suitable conditions the MSC would have to be modified.



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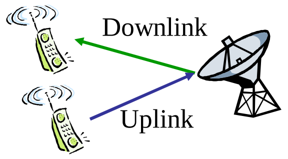
A non cooperative model of power control and transmission mode

The main shortcoming of this approach could be the intensity of uplink and downlink signaling, because this reduces the available resources.



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3. Non-cooperative approaches to Channel Allocation

3.1. Introduction

3.2. Non cooperative models of channel allocation

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Introduction

What do we mean for resource assignment in a wireless communications system?

When a user *i* asks for service, then a Base Station (BS) essentially answers with the following parameters:

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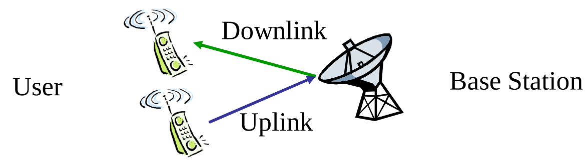


Introduction

• The recent evolution of mobile communication systems is being characterized by **high user expectations and demands in terms of quality of service** (QoS) provision.

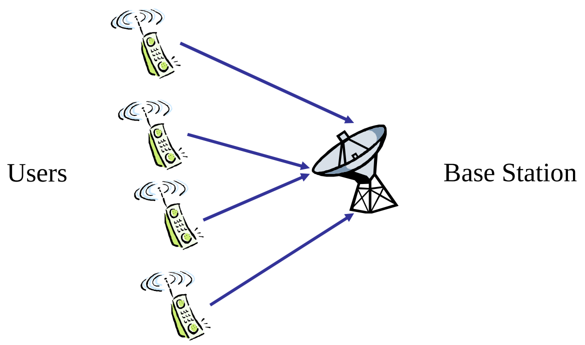
• Such demands require the design and implementation of the necessary means to accomplish an efficient use of the **scarce available resources**.

• One way to achieve such objective is through the development of **Radio Resource Management** (RRM) techniques.

• **An important RRM technique is channel assignment or allocation**. Channel assignment schemes are in charge of allocating, managing and distributing the available channels among users and services according to some QoS or system constraints.

Introduction

Radio resource management, and **channel allocation** in particular, are clear examples of **potential application fields for game theory** since their main functionality is to manage scarce radio resources among a set of competing users in a scenario where the actions of a particular user might affect the reminder users.



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Introduction

CHANNEL ALLOCATION ISSUES:

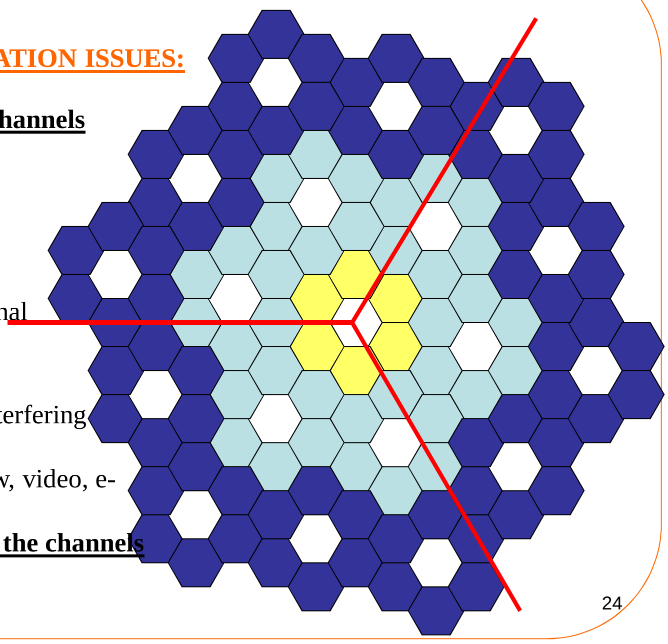
- **Limited number of channels**

- Interferencies

- Type of antennas
  - sectorized
  - omnidirectional

- Cells

- 1st and 2nd tiers of interfering cells
- Type of services (www, video, e-mail, calls, ...)
- **Homogeneous use of the channels in each cell.**



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Non cooperative models of channel allocation

AN EXAMPLE:

BS2

BS1		*C1*	*C2*
	C1	-0.104 / -0.104	0.608 / 0.608
	C2	0.608 / 0.608	-0.104 / -0.104

- Players: BSs
- Utility channel
  - Channel 0/1
  - Type of service
  - ...
- A third Nash Equilibrium ((1/2,1/2),(1/2,1/2)) with payoffs (0.252, 0.252).

Loss of efficiency: 58.55%

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Non cooperative models of channel allocation

Returning with the example:
What would happen if it is 100 times repeated?

Suppose 100 pairs of calls arrive successively to BS1 and BS2 with the same duration and needs. Consider, additionally, the utility obtained by a player (BS) is the average of the utility obtained in each call.

If coordination does not exist, every BS would obtain a utility of 0,252 with a loss of efficiency of 58,55 %.

If coordination does not exist but every BS assigns according to the past experiences, then every BS would obtain a utility of 0,601 with a loss of efficiency of only 1,17 %. (Gozálvez et al. 2004)

If coordination exists, then every BS would obtain a utility of 0,608 with a loss of efficiency of 0,00%. (Gozálvez et al. 2006)

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Non cooperative models of channel allocation

Wong & Wassell (2002) analyzed dynamic channel allocation (DCA) . They compared three different protocols to allocate channel: RND (Random channel allocation), LI (Least Interfered channel allocation) and GT (Least Interfered channel allocation with game theory).The first two methods are used in real networks.

The **RND** method simply consists of assigning the channel by random.

The **LI** method consists of assigning the **channel with the lowest interference** each time. In this method a **scan** is run after a number of transmitted frames in order to detect channel interefences.

The **GT** method follows the same idea as the LI method but **a non cooperative game is introduced in which the set of strategies is the number of frames transmitted before a new scan is run**. Of course, the utilities of the players (Access Points, AP) depend on the strategy profile. Using this method the probability to detect channel interefences is higher than in the LI method.

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Non cooperative models of channel allocation

Wong & Wassell (2002) show (by means of simulation) that the **GT method is better than the RND and the LI method** in all relevant parameters of QoS: 1-percentile SINR, $P_{>21dB}$, π_{avg}, T_0. $P_{>21dB}$ is the probabilitity of a packet being received with a SINR above 21dB, π_{avg} is the number of successful packets transmitted or received in average and T_0 is the overall throughput.

Therefore, using Game Theory in channel allocation seems a good idea in order to improve the efficiency of the wireless networks, and so the management of the (scarce) radio resource.

Nevertheless, in the example we observed that, apart from the classic concept of Nash equilibrium, something extra could be interesting. In this sense, Gozalvez et al (2004) study the possibility of **coordination** among the BSs. This coordination is carried out by taking simply into account the **past experiences** of channel allocation.

Thus, Gozalvez et al (2004) introduce a channel allocation method that assigns the channel with **better performance in the past** (**MinBLER**)

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Non cooperative models of channel allocation

AN EXAMPLE (cont.):

BS2

BS1	C1	C2
C1	-0.104 / -0.104	0.608 / 0.608
C2	0.608 / 0.608	-0.104 / -0.104

	RND		MinBLER	
	BS1	**BS2**	**BS1**	**BS2**
1	C1	C1	C1	C1
2	C1	C2	C2	C2
3	C2	C2	C2	C2
4	C2	C1	C1	C1
5	C1	C1	C1	C1
6	C2	C2	C2	C2
7	C1	C2	C1	C2
8	C2	C2	C1	C2
9	C2	C1	C1	C2
10	C2	C2	C1	C2

Non cooperative models of channel allocation

An important shortcoming of the MinBLER method is the usage of the different channels. From a maintenance point of view an homogenous usage of the channel in each BS would be convenient in order to save maintenance costs. For this reason, every certain time the system must be reseted.

A step further in the coordination of the BSs in order to improve the channel allocation is to consider the possibility of some **communication** among all BSs. This approach is described below.

Competition → **Coordination** → **Communication**

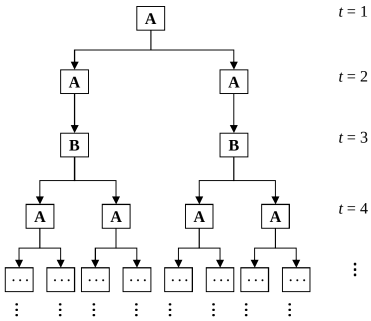
Wong & Wassell (2002) — Gozalvez et al (2004) — Gozalvez et al (2006)

Non cooperative models of channel allocation

Elements of the game:

0. Scenarios: S

EXAMPLE: Consider a system with two BSs (A and B) each with two channels. Denote by start-, end- or reject-call the possible events, a possible scenario is: start-call 1 in A, start-call 2 in A, start-call 3 in B, reject-call 4 in A, end-call 2, end-call 1, start-call 5 in A, end-call 3 and so on. From this information we can construct the following decision tree:



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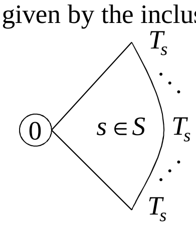
Non cooperative models of channel allocation

1. Players: $B = \{b_1, b_2, \ldots, b_n\}$ and nature $\{0\}$

Base stations (BSs) which are the responsible of the assignment channel decisions and nature chooses the scenario.

2. Decision tree:

- For each scenario $s \in S$ there is a tree T_s.
- For each T_s, $T_s(b_i)$ is the set of all nodes in which b_i has to make a channel decision.
- $T_s = \cup T_s(b_i)$ and $T_s(b_i) \cap T_s(b_j) = \varnothing$.
- $T_s^m(b_i)$ is the set of all nodes of $T_s(b_i)$ for which a channel decision has to be made before time m.
- The complete decision tree is given by the inclusion of nature which makes the scenario decision.



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Non cooperative models of channel allocation

3. Alternatives:

- C is the set of available channels. The alternatives set in each decision node is given by the number of non occupied channels at that moment.
- Given b_i and $j \in T_s(b_i)$ we denote by $C_s(b_i, j)$ his alternatives set in time j.

4. History for a node:

- Given a node t, history h_t is the information about the past which a BS knows when node t occurs.
- Denote by H_s the set of all possible histories for the scenario s.
- The information given by the histories determines the information sets of the game.
- Examples:
  - $h_t = \varnothing$ for all t.
  - h_t consists of the average utilities obtained till that moment in each of the used channels by the agents. (Gozálvez et al. (2004))
  - h_t consists of the channel allocations carried out by all BSs till that moment. (Gozálvez et al. (2006))

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Non cooperative models of channel allocation

5. Payoff function:

- For each $b_i \in B$ and each node $j \in T_s(b_i)$, the payoff function is given by

$$u_i : C_s(b_i, j) \times A_s(j) \to \mathbf{R}$$

- For each $h \in H_s$ we define the average payoff function for $b_i \in B$ as follows:

$$U_i(h) = \frac{1}{|T_s(b_i) \cap h|} \sum_{j \in T_s(b_i) \cap h} u_i(c(h_j), a_{-i}^j)$$

where $c(h_j) \in C_s(b_i, j)$ is the decision made by b_i in j according to h, h_j is the history for j according to h, $T_s(b_i) \cap h$ is the set of all nodes along h in which b_i had tomake a decision and a_{-i}^j is the channel allocations of all system except for i in instant j. If the history is infinite then payoff for each player is given by:

$$U_i(h) = \lim_{m \to +\infty} \frac{1}{|T_s^m(b_i) \cap h|} \sum_{j \in T_s^m(b_i) \cap h} u_i(c(h_j), a_{-i}^j)$$

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Non cooperative models of channel allocation

The game:

• For each $s \in S$ we define the game G_s by the 5-tuple:

$$\left(B, T_s, (T_s(b_i))_{b_i \in B}, (C_s(b_i, j))_{b_i \in B}^{j \in T_s(b_i)}, (u_i(\cdot,\cdot))_{b_i \in B}\right)$$

• And the game G is defined by the following 4-tuple: $\left(B \cup \{0\}, S, (G_s)_{s \in S}, P\right)$
where P is a distribution function defined over S.

Strategies:

• A strategy prescribes an action for each information set. Taking into account the information sets are determined by the history, we can say that for each different history the strategy prescribes an action (choose one of the available channels).

$$\sigma_i : H_i \to C$$

• Furthermore, we consider players can randomize, i.e, they can assign probabilities to each possible action, therefore we are considering the mixed extension of the game.

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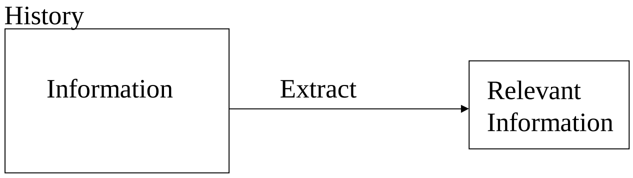
Non cooperative models of channel allocation

Now, What can we do?



• Let R be the set of all possible relevant information. Then an strategy (protocol of channel allocation), σ_i, is an action (mixed) for each $r \in R$.

$$\sigma_i : R \to \Delta C$$

•And the payoff in that instant is given by:

$$u_i(\sigma_i(r), a_{-i}) = \sum_{c \in C} p_c u_i(c, a_{-i})$$

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Non cooperative models of channel allocation

Defining strategies:

• Depending on the relevant information we can define different strategies. Considering the three previous examples on history information, we can define the following three strategies:

Relevant information

Number of channels used in BS

$$\sigma_i(r) = (p_c):\ p_c = \begin{cases} 0 & \text{if } c \text{ is occupied} \\ \frac{1}{k} & \text{otherwise} \end{cases}$$

Average utility of each channel in the past and number of used channels in BS

$$\sigma_i(r) = (p_c):\ p_c = \begin{cases} 0 & \text{if } c \notin \arg opt_c\ \{\overline{u_i(c)}\} \text{ and/or } c \text{ used} \\ \frac{1}{k} & \text{if } c \in \arg opt_c\ \{\overline{u_i(c)}\} \end{cases}$$

Used channels in each BS at that instant

$$\sigma_i(r) = (p_c):\ p_c = \begin{cases} 0 & \text{if } c \notin \arg opt_c\ \{u_i(c,r)\} \text{ and/or } c \text{ is used} \\ \frac{1}{k} & \text{if } c \in \arg opt_c\ \{u_i(c,r)\} \end{cases}$$

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Non cooperative models of channel allocation

Utility function:

• The utility function is based in parameters "easily" evaluable, in this particular case, we choose the interference level, in two ways, on the one hand the interference caused and on the second hand the interference received. Furthermore, the interference level is related to other basic parameters to evaluate the preformance of a communication system: throughput and transmission error rates.

• The "instant" utility is given by the following formula:

$$u_i(c,a_{-i}) = w\big(IR_1(c,a_{-i}) + \xi IR_2(c,a_{-i})\big) + (1-w)\big(IC_1(c,a_{-i}) + \xi IC_2(c,a_{-i})\big)$$

where the interference level can be exactly computed (Okumura-Hata) or approximately.

• If we take into account the type of service, then the above utility function can be modified by including this parameter in the evaluation..

$$u_i(c,a_{-i},t) = w(t)\big(IR_1(c,a_{-i}) + \xi IR_2(c,a_{-i})\big) + (1-w(t))\big(IC_1(c,a_{-i}) + \xi IC_2(c,a_{-i})\big)$$

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Non cooperative models of channel allocation

Comments:

- First, observe if the relevant information is the used channels in each BS, then we can change, without loss of generality, a_{-i} for r.
- Whe we refer to strategies we are refering to behaviour strategies and not to mixed strategies in the classical sense. We use this kind of strategies because it seems more adequate and reasonable for our allocation problem.
- The structure of the problem is very complex and thereby it is difficult to find equilibria for the game, in particular, subgame perfect equilibria (SPEs).
- The third strategy is based on a "forward induction principle" considering the channel allocation which optimezes the instant utility function taking into account the used channels in all BSs at that moment. On the other hand, the application of a "backward induction principle" does not seem reasonable, or it is very difficult to apply, given the structure of the problem.
- If the scenario is revealed, and the time horizon is finite, then it is known the existence of SPEs, but it does not mean that it is easy to find a simple and reasonable allocation protocol from the point of view of the telecommunication engineering.
- ...

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Non cooperative models of channel allocation

Some theoretical results:

Proposition 3 *Let be a mobile communication system with FCA, $s \in S$ a finite-term scenario for it and $G_s = (B, T_s, (T_s(i))_{i \in B}, C \cup \{*\}, (u_i(\cdot,\cdot))_{i \in B})$ be the associated $s-$channel allocation game. Then the forward strategy profile σ^f is a Nash equilibrium of the game. Furthermore it is a subgame perfect Nash equilibrium.*

Corollary 4 *Let be a mobile communication system with FCA and $G = (B \cup \{0\}, S, G_S, (p_s)_{s \in S})$ be the associated channel allocation game such that S is the set of all finite-term scenarios. Then the forward strategy profile σ^f is a Nash equilibrium of the game. Furthermore it is a subgame perfect Nash equilibrium.*

Proposition 5 *Let be a mobile communication system with FCA, $s \in S$ an infinite-term scenario for it and $G_s = (B, T_s, (T_s(i))_{i \in B}, C \cup \{*\}, (u_i(\cdot,\cdot))_{i \in B})$ be the associated $s-$channel allocation game. Then the forward strategy profile σ^f is a Nash equilibrium for each stage m of the game. Furthermore it is a subgame perfect Nash equilibrium for each stage m of the game.*

Corollary 6 *Let be a mobile communication system with FCA and $G = (B \cup \{0\}, S, G_S, (p_s)_{s \in S})$ be the associated channel allocation game. Then the forward strategy profile σ^f is a Nash equilibrium for each stage m of the game. Furthermore it is a subgame perfect Nash equilibrium for each stage m of the game.*

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Non cooperative models of channel allocation

Simulation results:

Throughput		Random	MinBLER		A2T, w=0.5		
		Perf. (kbps)	Perf. (kbps)	Impr. Random (%)	Perf. (kbps)	Impr. Random (%)	Impr. MinBLER (%)
System	Mean	18,23	18,64	2,19	19,2	5,05	2,91
	95%	13,33	14,02	4,92	15,3	12,87	8,36
WWW	Mean	18,57	18,89	1,69	19,39	4,22	2,57
	95%	13,78	14,34	3,9	15,43	10,69	7,06
Email	Mean	18.60	18,82	1,18	19,37	4,14	2,92
	95%	13.85	14,15	2,17	15,39	11,12	8,76
H263	Mean	17,53	18,2	3,68	18,84	6,95	3,39
	95%	12,37	13,59	8,97	15,08	17,97	9,88

Table 3. Throughput performance per services

Parameter	Random	A2T, w=0.5	A1T, w=0.5	E2T, w=0.5	A2T Selfish
% of H.263 video frames transmitted without delay	72.86	77.31	76.22	77.16	76.18
% of H.263 video frames without delay and BLER <= 5%	51.38	58.98	56.74	58.82	56.98

Table 4. H.263 video quality

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Non cooperative models of channel allocation

Simulation results:

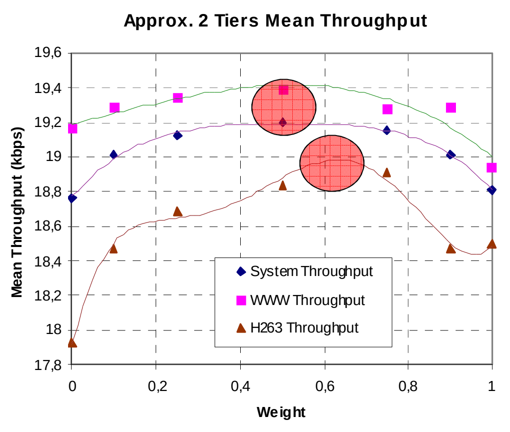

Figure 3. Effect of the weight parameter on the mean throughput performance

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Non cooperative models of channel allocation

Simulation results:

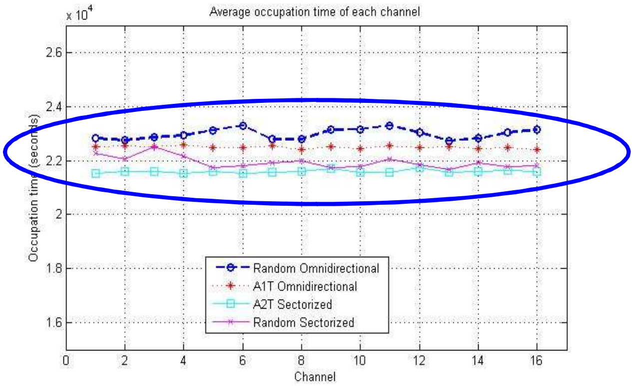

Figure 5. Average channel occupancy over the whole simulation time

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Non cooperative models of channel allocation

Conclusions and further research

• The simulation results show game theory can play a role in the context of telecommunication problems.

• The proposed algorithms are myopic, in the sense they use only the past and present information but not take into account what it can happen in the future.

• The proposed model opens a door to measure the maximum level of performance and so the possibility to know the efficiency of the algorithms (strategies) which can be proposed.

• Another alternative is to use a bayesian approach to define new algorithms (strategies).

• Simplify the game in order to be able to find equilibria.

• ...

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Non cooperative models of channel allocation

Further simulation results:

Parameter	Random	A2T, w=0.5	A1T, w=0.5	E2T, w=0.5	A2T Selfish
Mean throughput (kbps)	18,23	19,2	18,93	19,19	18,81
Minimum throughput guaranteed for 95% of the samples (kbps)	13,33	15,3	14,76	15,22	14,44
Mean normalized delay (ms/kbit)	61,65	58,66	59,47	58,66	59,84
Max normalized delay guaranteed for 95% of the samples (ms/kbit)	90,17	81,46	82,78	81,7	83,76
Mean BLER (%)	6,45	4,52	5,02	4,54	5,29
Maximum BLER guaranteed for 95% of the samples (%)	15,34	10,98	12,02	11,09	12,81
Mean number of CS changes per sec	2,25	1,83	1,96	1,82	2,03
Proportion of RLC blocks transmitted with the optimal CS (%)	72,20	79,19	76,96	78,75	76,19
Usage percentage of CS1 (%)	2,09	0,53	0,81	0,56	1,06
Usage percentage of CS2 (%)	3,51	1,51	2,01	1,56	2,29
Usage percentage of CS3 (%)	24,85	19,06	21,11	19,14	21,67
Usage percentage of CS4 (%)	69,55	78,90	76,07	78,75	74,98
Mode error of CS1 (%)	48,26	54,58	52,97	54,41	53,02
Mode error of CS2 (%)	83,87	84,47	84,19	84,40	84,58
Mode error of CS3 (%)	77,91	81,03	79,93	80,87	79,59
Mode error of CS4 (%)	6,46	4,82	5,31	4,82	5,42

Table 2. System performance comparison for sectorised networks

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Non cooperative models of channel allocation

Further simulation results:

Parameter	Random	minBLER	A1T	E1T	A2T
Mean throughput (kbps)	15,94	16,64	17,11	17,27	
Minimum throughput guaranteed for 95% of the samples (kbps)	8,63	9,66	10,82	11,26	
Mean normalized delay (ms/kbit)	70,75	68,05	66,04	65,26	
Max normalized delay guaranteed for 95% of the samples (ms/kbit)	119,79	114,55	105,93	100,85	
Mean BLER (%)	11,65	9,86	8,71	8,28	
Maximum BLER guaranteed for 95% of the samples (%)	31,71	26,74	22,26	20,71	
Mean number of CS changes per sec	2,75	2,61	2,54	2,51	
Proportion of RLC blocks transmitted with the optimal CS (%)	61,89	64,70	66,26	66,69	
Usage percentage of CS1 (%)	9,55	6,69	4,89	4,25	
Usage percentage of CS2 (%)	8,77	7,20	6,16	5,85	
Usage percentage of CS3 (%)	31,37	30,20	29,47	29,45	
Usage percentage of CS4 (%)	50,32	55,91	59,48	60,45	
Mode error of CS1 (%)	37,86	40,00	42,87	44,21	
Mode error of CS2 (%)	82,00	82,19	82,58	82,67	
Mode error of CS3 (%)	72,84	73,79	74,68	74,82	
Mode error of CS4 (%)	8,86	7,91	7,64	7,55	

Table 5. System performance comparison for sectorised networks

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Non cooperative models of channel allocation

Simulation parameters:

Parameter	Value
Cluster size	4
Cell radius	1km
Sectorisation (if aplicable)	120º
Modelled interference	1st and 2nd co-channel interfering tiers
Channels per sector	16
Users per sector	8
Traffic type	H.263 video: 2 users/sector WWW: 3 users/sector Email: 3 users/sector
Pathloss model	Okumura-Hata
Shadowing	Log-normal distribution. 6dB standard deviation and a 20m decorrelation distance
Fast Fading	Included through Look-Up Tables
ARQ protocol	Only for WWW and email users. Ack/Nack reports sent each 16 RLC blocks
LA updating period	100ms

Table 1. Simulation settings

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4. Game Theory and the business of sponsored search advertisements

4.1. Introduction and Motivation

4.2. Non cooperative model (2-players)

4.3. Auctions in the business of sponsored search advertisements

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Introduction and Motivation

• Internet has become the usual place for consumers to search for firms offering specific services.

TWO DRAWBACKS:

• Search can produce **irrelevant results** for the users.

• Search can produce **unstructured company listings**.

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Introduction and Motivation

A SOLUTION:

A NEW AUCTION MARKET:

Lim & Tang (2006) used the following example:

“*The GoTo search service offers firms two major benefits over the electronic Yellow Pages and other Internet search engines. First, since the firm pays GoTo the amount of its bid only when a consumer clicks on the firm's listing, GoTo offers pay-for-performance service (i.e., variable cost of advertising instead of fixed cost). Second, since the search results appear in descending order of the bid and since the bids are revealed to all firms, each firm can submit a new bid anytime so as to change the order at which it appears on the list.*”

to motivate the analysis of this new auction market.

Two real examples are the following:

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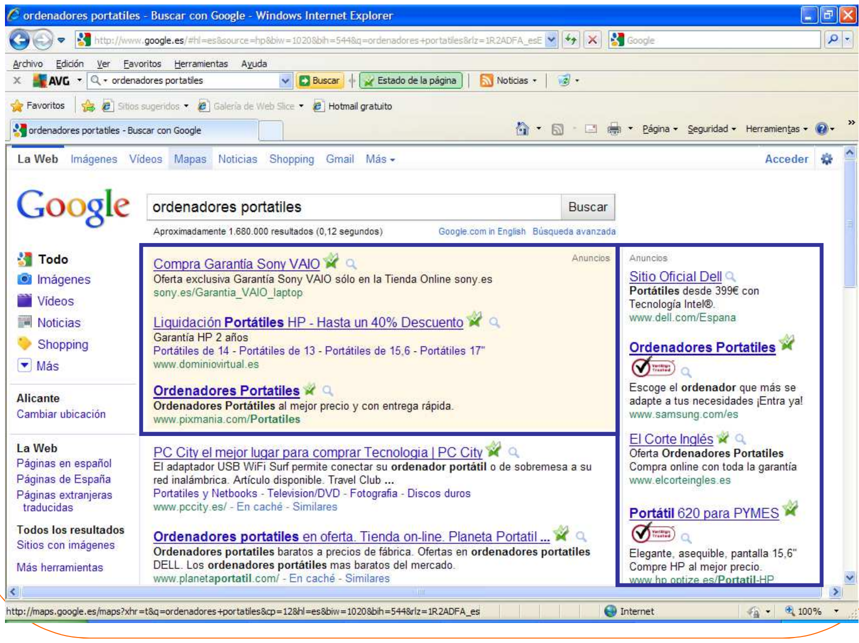
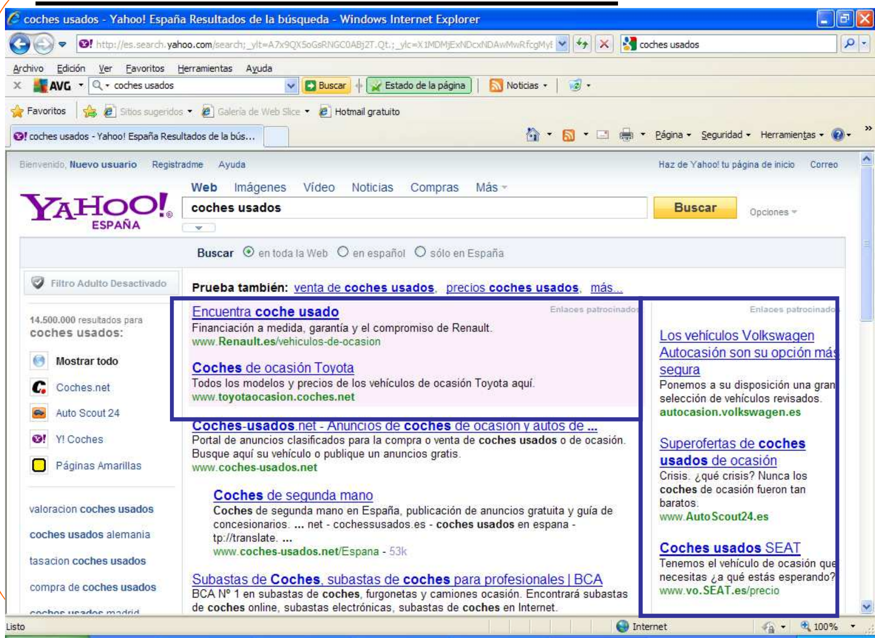
Introduction and Motivation



Introduction and Motivation



Introduction and Motivation

According to this we can consider the following:

A **ranking auction market** describes a situation in which a provider offers a service for ranking several firms (the bidders) in a search tool over the Internet, for example. This process enables the bidders to obtain more clicks on the name of their firms and in this way increase their incomes. **Each bidder is interested in reaching a position which is as high as possible in the ranking**, because the number of potential customers achieved depends on its status. The provider has not established an upper bound in the number of firms to be ranked. This means that all the bidders will be included in the ranking. Thus, we are considering a market situation with **one seller** (the provider) who owns as many different objects (the positions in the ranking) as the number of buyers (the bidders) who are interested in them.

Two possible approaches from Game Theory can be considered to tackle these situations. The first approach involves analyzing the problem from a competitive point of view (Lim & Tang, 2006). The second approach is to study these situations from a cooperative perspective, in which it is interesting to examine collusive behaviour (Aparicio et al. 2009).

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Non cooperative model (2-players)

In Lim and Tang (2006) they use a **non cooperative point** of view to develop a model with only **two firms** (bidders) which enables them to answer several questions related to:

• How can participating firms be induced to **bid aggressively**?

• How would the **rank-based response** (number of clicks the firm will receive when it is ranked first, second, etc.) affect a firm's bidding strategy?

The results are such that:

• A firm is more likely to bid aggressively when the rank-based response is highly **rank-sensitive** or when the profit resulting from each click is high.

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Non cooperative model (2-players)

Parameters to define the game in strategic form:

θ_1 : average profit per click for player I
θ_2 : average profit per click for player II
a : number of loyal customers
b : number of disloyal customers
l_1 : proportion of loyal customers to player I ($l_1 \geq 0$)
l_2 : proportion of loyal customers to player II ($l_2 \geq 0$)

$$l_1 + l_2 = 1$$

k : average number of click of disloyal customers ($0 \leq k \leq 2$)
p : number of clicks done for disloyal customers to the firm in the first position
α : minimum bid

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Non cooperative model (2-players)

(Lim & Tang, 2006)

II \ **I**	3α	2α	α
3α	$(\theta_1 - 3\alpha)(al_1 + b\frac{k}{2})$ $(\theta_2 - 3\alpha)(al_2 + b\frac{k}{2})$	$(\theta_1 - 3\alpha)(al_1 + bp)$ $(\theta_2 - 2\alpha)(al_2 + b(k - p))$	$(\theta_1 - 3\alpha)(al_1 + bp)$ $(\theta_2 - \alpha)(al_2 + b(k - b))$
2α	$(\theta_1 - 2\alpha)(al_1 + b(k - p))$ $(\theta_2 - 3\alpha)(al_2 + bp)$	$(\theta_1 - 2\alpha)(al_1 + b\frac{k}{2})$ $(\theta_2 - 2\alpha)(al_2 + b\frac{k}{2})$	$(\theta_1 - 2\alpha)(al_1 + bp)$ $(\theta_2 - \alpha)(al_2 + b(k - p))$
α	$(\theta_1 - \alpha)(al_1 + b(k - p))$ $(\theta_2 - 3\alpha)(al_2 + bp)$	$(\theta_1 - 2\alpha)(al_1 + b(k - p))$ $(\theta_2 - 3\alpha)(al_2 + bp)$	$(\theta_1 - \alpha)(al_1 + b\frac{k}{2})$ $(\theta_2 - \alpha)(al_2 + b\frac{k}{2})$

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The simplest version of the game

Non cooperative model (2-players)

(Lim & Tang, 2006)

I \ II	3α	2α	α
3α	$(\theta_1 - 3\alpha)(al_1 + b\frac{k}{2})$ $(\theta_2 - 3\alpha)(al_2 + b\frac{k}{2})$ **Strongly rank-sensitive**	$(\theta_1 - 3\alpha)(al_1 + bp)$ $(\theta_2 - 2\alpha)(al_2 + b(k - p))$	$(\theta_1 - 3\alpha)(al_1 + bp)$ $(\theta_2 - \alpha)(al_2 + b(k - b))$
2α	$(\theta_1 - 2\alpha)(al_1 + b(k - p))$ $(\theta_2 - 3\alpha)(al_2 + bp)$	$(\theta_1 - 2\alpha)(al_1 + b\frac{k}{2})$ $(\theta_2 - 2\alpha)(al_2 + b\frac{k}{2})$	$(\theta_1 - 2\alpha)(al_1 + bp)$ $(\theta_2 - \alpha)(al_2 + b(k - p))$
α	$(\theta_1 - \alpha)(al_1 + b(k - p))$ $(\theta_2 - 3\alpha)(al_2 + bp)$	$(\theta_1 - 2\alpha)(al_1 + b(k - p))$ $(\theta_2 - 3\alpha)(al_2 + bp)$	$(\theta_1 - \alpha)(al_1 + b\frac{k}{2})$ $(\theta_2 - \alpha)(al_2 + b\frac{k}{2})$ **Weakly rank-sensitive**

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The simplest version of the game

Non cooperative model (2-players)

The final result depends on two main elements:

1. The unitary profit per click.

2. The rank-sensitiveness of the list.

Lim & Tang (2006) use a probabilistic model in which the bidders have different expectations on the number of clicks received in the first position. In this sense, they also analyze how the information provided by the search service could induce bidders to bid more aggressively.

Similar results are obtained by simulation in Aparicio et al (2010) for more than two bidders.

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Non cooperative model (2-players)

Parameters to define the game in strategic form in the probabilistic model:

θ_1 : average profit per click for player I
θ_2 : average profit per click for player II
a : number of loyal customers
b : number of disloyal customers
l_1 : proportion of loyal customers to player I ($l_1 \geq 0$)
l_2 : proportion of loyal customers to player II ($l_2 \geq 0$) $l_1 + l_2 = 1$

k : average number of click of disloyal customers ($0 \leq k \leq 2$)
$p \approx \overline{p} + N(0,V)$ random variable of the number of clicks done for disloyal customers to the firm in the first position
$f_i \approx p + N(0,s_i)$ random variable of the private forecast of player i on the number of clicks done in the first position

$$E(p|f_i) = (1-t_i)\overline{p} + t_i f_i, \text{ where } t_i = \frac{V}{V+s_i}$$

α : minimum bid

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Non cooperative model (2-players)

(Lim & Tang, 2006)

II \ I	3α	2α	α
3α	$(\theta_1-3\alpha)(al_1+b\frac{k}{2})$ $(\theta_2-3\alpha)(al_2+b\frac{k}{2})$	$(\theta_1-3\alpha)(al_1+b\overline{p}+bt_1(f_1-\overline{p}))$ $(\theta_2-2\alpha)(al_2+bk-b\overline{p}-bt_2d_1(f_1-\overline{p}))$	$(\theta_1-3\alpha)(al_1+b\overline{p}+bt_1(f_1-\overline{p}))$ $(\theta_2-\alpha)(al_2+bk-b\overline{p}-bt_2d_1(f_1-\overline{p}))$
2α	$(\theta_1-2\alpha)(al_1+bk-b\overline{p}-bt_1(f_1-\overline{p}))$ $(\theta_2-3\alpha)(al_2+b\overline{p}+bt_2d_1(f_1-\overline{p}))$	$(\theta_1-2\alpha)(al_1+b\frac{k}{2})$ $(\theta_2-2\alpha)(al_2+b\frac{k}{2})$	$(\theta_1-2\alpha)(al_1+b\overline{p}+bt_1(f_1-\overline{p}))$ $(\theta_2-\alpha)(al_2+bk-b\overline{p}-bt_2d_1(f_1-\overline{p}))$
α	$(\theta_1-\alpha)(al_1+bk-b\overline{p}-bt_1(f_1-\overline{p}))$ $(\theta_2-3\alpha)(al_2+b\overline{p}+bt_2d_1(f_1-\overline{p}))$	$(\theta_1-2\alpha)(al_1+bk-b\overline{p}-bt_1(f_1-\overline{p}))$ $(\theta_2-3\alpha)(al_2+b\overline{p}+bt_2d_1(f_1-\overline{p}))$	$(\theta_1-\alpha)(al_1+b\frac{k}{2})$ $(\theta_2-\alpha)(al_2+b\frac{k}{2})$

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From the point of view of player I

Non cooperative model (2-players)

(Lim & Tang, 2006)

For each bidder there are four critical points. From these points it is possible to derive the best response function for each bidder and to analyze when the bidders will bid more aggressively.

- For $\theta_i > 3\alpha$, α is a dominant strategy for bidder i if and only if $f_i < \underline{p} + \gamma_{4i}$.

- $(3\alpha, 3\alpha)$ will be the only Bayesian Nash Equilibrium if and only if
  (1) the private forecasts of p are high enough,
  (2) but within the same range, and
  (3) the margins of the firms are not too different but higher than 3α.

$$\gamma_{1i} = \left(\frac{2\alpha a l_i}{b(\theta_i - \alpha)} + \frac{\theta_i + \alpha}{\theta_i - \alpha}\frac{k}{2} - \bar{p}\right)\left(1 + \frac{s_i}{V}\right)$$

$$\gamma_{2i} = \left(\frac{\alpha a l_i}{b(\theta_i - 3\alpha)} + \frac{\theta_i - 2\alpha}{\theta_i - 3\alpha}\frac{k}{2} - \bar{p}\right)\left(1 + \frac{s_i}{V}\right)$$

$$\gamma_{3i} = \left(\frac{\alpha a l_i}{b(\theta_i - 2\alpha)} + \frac{\theta_i - \alpha}{\theta_i - 2\alpha}\frac{k}{2} - \bar{p}\right)\left(1 + \frac{s_i}{V}\right)$$

$$\gamma_{4i} = \left(\frac{\alpha a l_i}{b(\theta_i - \alpha)} + \frac{\theta_i}{\theta_i - \alpha}\frac{k}{2} - \bar{p}\right)\left(1 + \frac{s_i}{V}\right)$$

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Auctions in the business of sponsored search advertisements

DESIRABLE PROPERTIES OF AN (MULTI-OBJECT) AUCTION:

From the **point of view of bidders**, an auction would be reasonable if it satisfies at least the following two characteristics:

- bidders with the highest bids get the items (**fairness**)
- winner bidders do not pay for the gained objects more than they are actually willing to pay for them (**no winner’s curse**).

From the **auctioneer perspective**:

- providing her the highest expected revenue (**optimality**)
- assigning the objects to who most highly value them (**efficiency**).

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Auctions in the business of sponsored search advertisements

DESCRIPTION OF THE MODEL (I):

- **Three advertising positions** $A < B < C$ such that the ratio between their number of clicks is $\#B/\#A = \alpha \in (0, 1)$ and $\#C/\#A = \beta \in (0, \alpha)$.

- n > 3 **risk-neutral bidders** who want to get just one of these positions.

- We assume that **the search engine announces (reveals) the number of clicks that will be received in the first, second and third position**. We consider that **the number of clicks received in the first position is normalized into 1**.

- The **valuation** for each position of bidder i is θ_i, $\alpha\theta_i$ and $\beta\theta_i$, respectively. Each type $\theta_i \in [0, 1]$, which is information only known for bidder i, is an independent realization of a continuous random variable Θ with c.d.f. F and density function f.

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Auctions in the business of sponsored search advertisements

DESCRIPTION OF THE MODEL (and II):

- **Each bidder simultaneously and independently submits a single bid** $b_i \in [0, 1]$ specifying the maximum unit-price offer at which they are willing to pay. That is, bidder i would pay at most b_i for the position A, would pay at most αb_i for the position B and would pay at most βb_i for the position C.

- A **strategy** for bidder i is a function $b_i(.)$ that assigns a bid $b_i(\theta_i)$ to each type of bidder θ_i.

- We assume, given the symmetry of the model, that $b_i(\theta_i) = b(\theta_i)$ is the bid function used in equilibrium by all bidders, where $b(\theta_i)$ is a strictly monotone and differentiable function.

- The objects are allocated based on the rank of the bids. The price paid by each bidder will depend on the auction mechanism adopted by the auctioneer. All aspects of **this model** and the **auction mechanism** chosen are assumed to be **common knowledge**.

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Auctions in the business of sponsored search advertisements

SOME WELL-KNOWN AND CLASSICAL AUCTION MECHANISMS:

• The **Uniform-Price Auction** (**UP**): All winner bidders pay the lowest accepted bid.

• The **Discriminatory-Price Auction** (**DP**): Every winner bidder pays his own bid.

• The **Vickrey Auction** (**VI**): Each winner bidder pays the bid of the bidder that she displaces (i.e., the price of the object sold if she removes her bid).

• The **Generalized Second Price Auction** (**GSP**): Each winner bidder pays for a click in his link the bid submitted by the bidder below him in the ordered list.

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Auctions in the business of sponsored search advertisements

A PARAMETRIC FAMILY OF AUCTION MECHANISMS (I):

Given a set of n bids, $b = \{b_1,\ldots, b_n\}$, b_{-i} is the set of all bids except bid i, $b^{(i)}$ is the i-th–highest bid of the set b. Bidder i must pay the following:

$$\begin{cases} \gamma_1 b_i + (1-\alpha+\gamma_2-\gamma_1)b_{-i}^{(1)} + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(1)} < b_i \\ \gamma_2 b_i + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(2)} < b_i < b_{-i}^{(1)} \\ \gamma_3 b_i + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(3)} < b_i < b_{-i}^{(2)} \\ 0 & \text{otherwise} \end{cases}$$

If α=1 and β=1, then we face the classical auction for three identical objects. Thus

• If $\gamma_1 = 1$, $\gamma_2 = 1$ and $\gamma_3 = 1$, then we obtain the discriminatory-price auction (DP).
• If $\gamma_1 = 0$, $\gamma_2 = 0$ and $\gamma_3 = 1$, then we obtain the uniform-price auction (UP).
• If $\gamma_1 = 0$, $\gamma_2 = 0$ and $\gamma_3 = 0$, then we obtain the Vickrey auction (VI).

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Auctions in the business of sponsored search advertisements

A PARAMETRIC FAMILY OF AUCTION MECHANISMS (II):

Given a set of n bids, $b = \{b_1,\ldots, b_n\}$, b_{-i} is the set of all bids except bid i, $b^{(i)}$ is the i-th–highest bid of the set b. Bidder i must pay the following:

$$\begin{cases} \gamma_1 b_i + (1-\alpha+\gamma_2-\gamma_1)b_{-i}^{(1)} + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(1)} < b_i \\ \gamma_2 b_i + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(2)} < b_i < b_{-i}^{(1)} \\ \gamma_3 b_i + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(3)} < b_i < b_{-i}^{(2)} \\ 0 & \text{otherwise} \end{cases}$$

If $\alpha\in(0, 1)$ and $\beta\in(0, \alpha)$, then we face an auction for three commonly ranked objects but not identical in general. Thus

- If $\gamma_1 = 1$, $\gamma_2 = \alpha$ and $\gamma_3 = \beta$, then we obtain a generalization of the discriminatory-price auction (**PDP**).
- If $\gamma_1 = 0$, $\gamma_2 = 0$ and $\gamma_3 = \beta$, then we obtain a generalization of the uniform-price auction (**PUP**).
- If $\gamma_1 = \gamma_2 = \gamma_3 = 0$, then we obtain a generalization of the Vickrey auction (**PVI**).

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Auctions in the business of sponsored search advertisements

A PARAMETRIC FAMILY OF AUCTION MECHANISMS (III):

Given a set of n bids, $b = \{b_1,\ldots, b_n\}$, b_{-i} is the set of all bids except bid i, $b^{(i)}$ is the i-th–highest bid of the set b. Bidder i must pay the following:

$$\begin{cases} \gamma_1 b_i + (1-\alpha+\gamma_2-\gamma_1)b_{-i}^{(1)} + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(1)} < b_i \\ \gamma_2 b_i + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(2)} < b_i < b_{-i}^{(1)} \\ \gamma_3 b_i + (\beta-\gamma_3)b_{-i}^{(3)} & \text{if } b_{-i}^{(3)} < b_i < b_{-i}^{(2)} \\ 0 & \text{otherwise} \end{cases}$$

If $\gamma_1\in[0, 1-\alpha+\gamma_2]$, $\gamma_2\in[0, \alpha-\beta+\gamma_3]$ and $\gamma_3\in[0, \beta]$, then we avoid the winning bidders to pay more than their valuations of the objects.

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Auctions in the business of sponsored search advertisements

THE GENERALIZED SECOND PRICE AUCTION (GSP):

Given a set of n bids, $b = \{b_1,\dots, b_n\}$, b_{-i} is the set of all bids except bid i, $b^{(i)}$ is the i-th–highest bid of the set b. Bidder i must pay the following:

$$\begin{cases} b_{-i}^{(1)} & if\ b_{-i}^{(1)} < b_i \\ \alpha b_{-i}^{(2)} & if\ b_{-i}^{(2)} < b_i < b_{-i}^{(1)} \\ \beta b_{-i}^{(3)} & if\ b_{-i}^{(3)} < b_i < b_{-i}^{(2)} \\ 0 & otherwise \end{cases}$$

This auction mechanism is simpler than those in the parametric family, but as we will see, auction mechanisms in the parametric family have better properties than GSP.

Auctions in the business of sponsored search advertisements

ARE GSP, VI AND PVI THE SAME ? (I)

First we show how to compute VI for multi-object auctions. Bidder i must pay the following:

- Let v_i denote the value that i gets from the efficient allocation.
- Let V denote the total surplus from the efficient allocation ($V = \Sigma_i\ v_i$).
- Let V_{-i} denote the total surplus that could be generated if i did not participate (and the seller allocated efficiently the goods among the other bidders).
- Let $p_i = v_i - (V - V_{-i})$ be is the Vickrey payment.

Auctions in the business of sponsored search advertisements

ARE GSP, VI AND PVI THE SAME ? (II)

EXAMPLE: Suppose there are three positions, that get 1, 0.6 and 0.3 of a bundle of 300 clicks, respectively (hence $\alpha = 0.6$ and $\beta = 0.3$). There are four bidders (B1, B2, B3 and B4) with click bids 0.8, 0.6, 0.4 and 0.2, respectively.

Auction	VI		GSP		PVI	
	Position	**Pays**	**Position**	**Pays**	**Position**	**Pays**
B1	1	0.42	1	0.60	1	0.42
B2	2	0.18	2	0.24	2	0.18
B3	3	0.06	3	0.06	3	0.06
B4	4	0.00	4	0.00	4	0.00

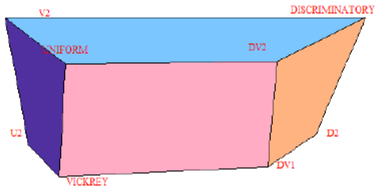
Auctions in the business of sponsored search advertisements

A PARAMETRIC FAMILY OF AUCTION MECHANISMS (IV):

Given a set of n bids, $b = \{b_1,\ldots, b_n\}$, b_{-i} is the set of all bids except bid i, $b^{(i)}$ is the i-th–highest bid of the set b. Bidder i must pay the following:

$$\begin{cases} \gamma_1 b_i + (1-\alpha+\gamma_2-\gamma_1)b_{-i}^{(1)} + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(1)} < b_i \\ \gamma_2 b_i + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(2)} < b_i < b_{-i}^{(1)} \\ \gamma_3 b_i + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(3)} < b_i < b_{-i}^{(2)} \\ 0 & otherwise \end{cases}$$

where $\gamma_1 \in [0, 1-\alpha+\gamma_2]$, $\gamma_2 \in [0, \alpha-\beta+\gamma_3]$ and $\gamma_3 \in [0, \beta]$.



Auctions in the business of sponsored search advertisements

A PARAMETRIC FAMILY OF AUCTION MECHANISMS (V):

Given a set of n bids, $b = \{b_1,\ldots, b_n\}$, b_{-i} is the set of all bids except bid i, $b^{(i)}$ is the i-th–highest bid of the set b. Bidder i must pay the following:

$$\begin{cases} \gamma_1 b_i + (1-\alpha+\gamma_2-\gamma_1)b_{-i}^{(1)} + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(1)} < b_i \\ \gamma_2 b_i + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(2)} < b_i < b_{-i}^{(1)} \\ \gamma_3 b_i + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(3)} < b_i < b_{-i}^{(2)} \\ 0 & otherwise \end{cases}$$

where $\gamma_1 \in [0, 1-\alpha+\gamma_2]$, $\gamma_2 \in [0, \alpha-\beta+\gamma_3]$ and $\gamma_3 \in [0, \beta]$.

From the **point of view of bidders**, these auctions satisfy the following two characteristics:

- bidders with the highest bids get the items (**fairness**)
- winner bidders do not pay for the gained objects more than they are actually willing to pay for them (**no winner's curse**).

Auctions in the business of sponsored search advertisements

A PARAMETRIC FAMILY OF AUCTION MECHANISMS (VI):

Theorem 1

If an auction mechanism M belonging to the family is used, then the unique symmetric Bayesian Nash equilibrium $(b^*(\theta_1), b^*(\theta_2), \ldots, b^*(\theta_n))$, with the condition $\lim_{\theta_i \to 0} b^*(\theta_i) = 0$, is efficient and given by:

If $\gamma_1 = \gamma_2 = \gamma_3 = 0$, $b^*(\theta_i) = \theta_i$. ← **TRUTHTELLING**

And in other case

$$b^*(\theta_i) = \theta_i - \frac{\int_0^{\theta_i} e^{\int a(t)dt} f(t)dt}{e^{\int a(\theta_i)d\theta_i}}$$

where

$$a(\theta_i) = \frac{\left(\frac{-2+2\alpha(n-1)-(n-1)(n-2)\beta}{2(n-2)(n-3)}F^2(\theta_i) + \frac{-\alpha+(n-2)\beta}{n-3}F(\theta_i) - \frac{\beta}{2}\right)f(\theta_i)}{\frac{-2\gamma_1+2\gamma_2(n-1)-(n-1)(n-2)\gamma_3}{2(n-1)(n-2)(n-3)}F^3(\theta_i) + \frac{-\gamma_2+(n-2)\gamma_3}{(n-2)(n-3)}F^2(\theta_i) - \frac{\gamma_3}{2(n-3)}F(\theta_i)}.$$

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A PARAMETRIC FAMILY OF AUCTION MECHANISMS (VII):

Theorem 2: Revenue Equivalence.

The expected payment for the bidders and the expected revenue for the auctioneer are, respectively,

$$\begin{aligned} P_i(\theta_i) = & \left(1-(n-1)\alpha+\frac{1}{2}(n-1)(n-2)\beta\right)\left(\theta_i F^{n-1}(\theta_i)-\int_0^{\theta_i} F^{n-1}(t)dt\right) \\ & +((n-1)\alpha-(n-1)(n-2)\beta)\left(\theta_i F^{n-2}(\theta_i)-\int_0^{\theta_i} F^{n-2}(t)dt\right) \\ & +\frac{1}{2}(n-1)(n-2)\beta\left(\theta_i F^{n-3}(\theta_i)-\int_0^{\theta_i} F^{n-3}(t)dt\right), \end{aligned}$$

$$\begin{aligned} P_{AUC} = & n\int_0^1 P_i(\theta_i)f(\theta_i)d\theta_i = 1+\alpha+\beta \\ & +(n-1)\left(1-(n-1)\alpha+\frac{1}{2}(n-1)(n-2)\beta\right)\int_0^1 F^n(\theta_i)d\theta_i \\ & +\left(-n+n(2n-3)\alpha-\frac{1}{2}n(3n-5)(n-2)\beta\right)\int_0^1 F^{n-1}(\theta_i)d\theta_i \\ & +\left(-n(n-1)\alpha+\frac{1}{2}n(n-1)(3n-7)\beta\right)\int_0^1 F^{n-2}(\theta_i)d\theta_i \\ & -\frac{1}{2}n(n-1)(n-2)\beta\int_0^1 F^{n-3}(\theta_i)d\theta_i. \end{aligned}$$

Auctions in the business of sponsored search advertisements

GSP vs AUCTION MECHANISMS IN THE PARAMETRIC FAMILY

- GSP auction does not belong to the parametric family (it is not possible to find a combination of parameters γ1, γ2 and γ3 to obtain the GSP).

- Gomes and Sweeney (2012) proved that the GSP auction may not have an efficient symmetric Bayesian Nash equilibrium but all auctions in the parametric family have.

- Furthermore when there are efficient equilibria in the GSP auction then their expected revenue coincide with PAUC in Theorem 2.

- GSP is the auction mechanism used by Google and Yahoo in their sponsored search advertisement auctions.

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Auctions in the business of sponsored search advertisements

A PARAMETRIC FAMILY OF AUCTION MECHANISMS (and VIII):

Given a set of n bids, $b = \{b_1,\ldots, b_n\}$, b_{-i} is the set of all bids except bid i, $b^{(i)}$ is the i-th–highest bid of the set b. Bidder i must pay the following:

$$\begin{cases} \gamma_1 b_i + (1-\alpha+\gamma_2-\gamma_1)b_{-i}^{(1)} + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(1)} < b_i \\ \gamma_2 b_i + (\alpha-\beta+\gamma_3-\gamma_2)b_{-i}^{(2)} + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(2)} < b_i < b_{-i}^{(1)} \\ \gamma_3 b_i + (\beta-\gamma_3)b_{-i}^{(3)} & if\ b_{-i}^{(3)} < b_i < b_{-i}^{(2)} \\ 0 & otherwise \end{cases}$$

where $\gamma_1 \in [0, 1-\alpha+\gamma_2]$, $\gamma_2 \in [0, \alpha-\beta+\gamma_3]$ and $\gamma_3 \in [0, \beta]$.

From the **auctioneer perspective**, these auctions satisfy the following two characteristics:

- provide her the highest expected revenue (**optimality**).
- assign the objects to who most highly value them (**efficiency**).

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