Socially-Aware and Cooperative Network Design Games

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The talk is based on joint works with
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Shapley network design game

Our point of departure is Shapley network design game introduced in

E. Anshelevich, A. Dasgupta, J. Kleinberg, E. Tardos, T. Wexler and T. Roughgarden, "The price of stability for network design with fair cost allocation", in Proceedings of FOCS'04, pp.295-304, Rome, October 2004.

Shapley network design game

The game is played on a directed graph $G = (V, E)$, where each player $i \in I = \{1, 2, \dots k\}$ is identified with a source-sink pair (s_i, t_i).

Every player i picks a path S_i from its source to its destination.

We will refer to the path S_i also as the strategy chosen by player i.

Let x_e denote the number of paths that go through edge e and c_e denote the cost of edge e.

Shapley network design game

The cost of network formation is distributed as follows:

If edge e lies in x_e of the chosen paths, then each player choosing such an edge pays a proportional share $\pi_e = \frac{c_e}{x_e}$ of the cost. A player pays for all the links that he/she uses.

Therefore, the objective function J^i that user i wants to minimize is given by

$$J^i = \sum_{e \in S_i} \pi_e. \quad (1)$$

The game achieves Nash equilibrium in pure strategies.

Shapley network design game

It has been shown by Anshelevich et. al. that the Price of Anarchy (the ratio between the cost of the worst stable network and the cost of the optimal network) can be

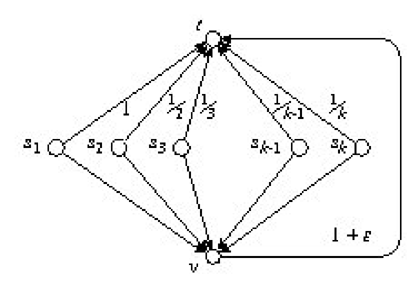
$$PoA = k,$$

and the price of stability (the ratio between the best stable network and the cost of the optimal network) can be

$$PoS = O(ln(k)).$$

Motivating example

Let us consider the following example:



Socially-Aware Network Design Game

The main advantage of game based network formation:

distributed network management

The main advantage of the centralized network planning

network cost optimization

We would like to find an approach which combines the above positive features.

Socially-Aware Network Design Game

Firstly, we suggest to introduce a social cost component into the players' objectives

$$J^i = \sum_{e \in S_i} \pi_e + \alpha \sum_{e \in \cup_j S_j} c_e \qquad (2)$$

Note that for $\alpha = 0$ we retrieve the original Shapley Network Design Game.

SAND Game properties

We have established the following properties of the SAND game:

- It is a potential game with the following potential function

$$\Phi(S) = \sum_{e \in E} \sum_{x=1}^{x_e} \frac{c_e}{x} + \alpha \sum_{e \in \cup_j S_j} c_e \qquad (3)$$

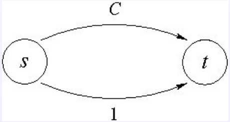
- The Price of Anarchy could be as high as $k(1+\alpha)$, where k is the number of players.

SAND Game properties

- The Price of Stability is upper bounded by $(\alpha + H_k)/(\alpha + 1)$, where $H_k = \sum_{i=1}^{k} 1/i$ is the k-th harmonic number.
- The Reachable Price of Anarchy (achieved by best response starting from empty network) is upper bounded by $k(\alpha + 1)/(\alpha + 1/k)$.

SAND Game properties

Consider an example:



If all the players start at link with cost C and $\alpha \geq \frac{C}{k} - 1$, no player has a gain to deviate and choose the link with cost equal to 1. Then, the cost of the network is C.

Network Administrator Driven Socially-Aware Network Design Game

To avoid being stuck in not efficient equilibria, we propose a Stackelberg type modification of the SAND game.

In NAD-SAND game, a network administrator plays before the users and his/her aim is to drive the players to the best Nash equilibrium.

Network Administrator Driven Socially-Aware Network Design Game

Since computing the optimal Stackelberg strategy is NP-hard, we present a simple strategy that achieves consistent performance improvements.

1. The network administrator solves a generalized minimum spanning tree problem, determining the minimum-cost subnetwork such that the source / destination nodes of each player are connected by a path. Let E^{opt} be such subnetwork.
2. The network administrator chooses all links belonging to E^{opt}, thus offering to share eventually their cost with the other players.
3. At this point, all the k users play the SAND game, trying to optimize their own objective function (2).

Network Administrator Driven Socially-Aware Network Design Game

We observe that the first step of the NAD-SAND game involves solving an NP-Complete problem.

However, several efficient heuristics and approximation algorithms have been proposed to solve such problem in a reasonable computation time.

We also observe that as $\alpha \to \infty$, the NAD-SAND game always reaches the minimum cost network since for each player the cost of choosing any link that does not belong to the minimum-cost subnetwork (i.e., to E^{opt}) has an exceedingly large cost.

Thus, $PoA \to 1$ as $\alpha \to \infty$ in the NAD-SAND game.

Cooperative game introduction

There are some drawbacks in socially-aware approach:

- SAND game still can be not really efficient and NAD-SAND game requires additional effort from the network administrator.
- Not everybody is eager to pay a social related cost.
- Even if the social component is later redistributed; it is not clear how.

Cooperative game approach appears to provide answers to the above questions.

Shapley value

The Shapley value is a widely applied concept for solving cooperative games.

It is a possible way to allocate the total costs among the members of a coalition, taking into account their different contributions for the coalition.

The main advantage of the Shapley value is that it provides a solution that is both unique and fair.

Shapley value

A Shapley function ϕ is a function that assigns to each possible characteristic function v a vector of real numbers, i.e.,

$$\phi(v) = [\phi_1(v), \ldots, \phi_i(v), \ldots, \phi_n(v)], \qquad (4)$$

where $\phi_i(v)$ represents the cost of player i in the game.

The characteristic function, v, is a real-valued function that associates with every non-empty subset $\mathcal{S}$ of $\mathcal{I}$ (i.e., a coalition) a real number $v(\mathcal{S})$, the cost of $\mathcal{S}$, with the following properties:

1. $v(\emptyset) = 0$.
2. (Subadditivity) if $\mathcal{S}$ and $\mathcal{T}$ are disjoint coalitions ($\mathcal{S} \cap \mathcal{T} = \emptyset$), then $v(\mathcal{S}) + v(\mathcal{T}) \geq v(\mathcal{S} \cup \mathcal{T})$.

Shapley value

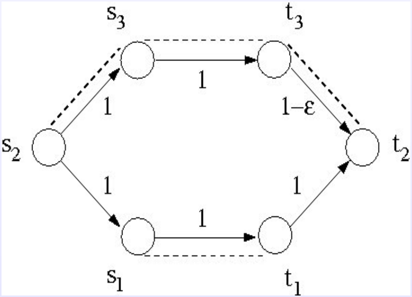
Some "natural" candidate characteristic functions for our game:

1. Players in $\mathcal{S}$ and players in $\mathcal{I} - \mathcal{S}$ form two separate coalitions. Each coalition tries to minimize the total cost for its members, taking into account the selfish behavior of the other coalition. A Nash equilibrium is reached, and $v(\mathcal{S})$ is defined as the total cost for members in $\mathcal{S}$ at this equilibrium.
2. The value of the coalition $\mathcal{S}$ is defined as its security level, i.e. as the minimum total cost that $\mathcal{S}$ can guarantee to itself when members in $\mathcal{I} - \mathcal{S}$ act collectively in order to maximize the cost for the coalition $\mathcal{S}$.
3. The value of coalition $\mathcal{S}$ is equal to the minimum cost that its members would incur if players in $\mathcal{I} - \mathcal{S}$ would be absent.

Shapley value

We note that, in our specific game, these three definitions give increasing value to a coalition $\mathcal{S}$.

To better illustrate the differences underlying the definitions of characteristic function, let us consider the hexagon network scenario.



Shapley value

Table: Hexagon network scenario: characteristic function values, $v(\mathcal{S})$, for definitions (1), (2) and (3).

	Characteristic Function candidates ($v(\mathcal{S})$)		
Coalition ($\mathcal{S}$)	Definition (1)	Definition (2)	Definition (3)
$\emptyset$	0	0	0
1	1	1	1
2	2.5-ϵ	2.5 - ϵ	3-ϵ
3	0.5	1	1
12	3	3	3
13	1.5	1.5	2
23	3-ϵ	3-ϵ	3-ϵ
123	4-ϵ	4-ϵ	4-ϵ

Shapley value

We have the following general result:

Theorem

In the Cooperative Network Formation Game, the security level and the minimal cost of coalition satisfy the axioms of characteristic function.

Shapley value

However, computing the Shapley value is an NP-complete problem.

$$\phi_i(v) = \sum_{S \subset I \setminus \{i\}} \frac{|S|!(k-|S|-1)!}{k!} \left(v(S \cup \{i\}) - v(S)\right)$$

Nash bargaining solution

Let u_i denote the maximal acceptable cost that user i is willing to pay.

In the present work we suggest the following three options:

1. the cost for user i to connect its source-destination nodes in a purely non-cooperative game (i.e., the Nash equilibrium solution);
2. the cost for user i to connect its source-destination nodes in a zero-sum game where all the other players are trying to maximize the cost of user i;
3. the cost for user i to connect its source-destination nodes when there is no other player.

Nash bargaining solution

The vector $u = \{u_1, u_2, \dots u_n\}$ is also called the disagreement point of the cooperative network formation game (i.e., what will happen if players cannot come to an agreement).

Clearly, the cost achieved by every player at any agreement point (every possible outcome of the bargaining game) has to be at most equal to the cost achieved at the disagreement point.

The Nash bargaining solution is given by the following optimization problem:

$$\max_{\alpha_i} \prod_{i=1}^{n} (u_i - \alpha_i) \tag{5}$$

$$\text{s.t.} \sum_{i=1}^{n} \alpha_i = u_{soc} \tag{6}$$

Nash bargaining solution

and can be given in explicit form:

Theorem

The Nash bargaining solution for player i, α_i is given by the following expression:

$$\alpha_i = u_i - \frac{\sum_k u_k - u_{soc}}{m}, \tag{7}$$

where m coincides with the number of players n (i.e., $m \equiv n$) if we allow for negative costs (i.e., some α_i values are negative, which means that some players are actually paid to ensure their participation).

Nash bargaining solution

Theorem

If only non-negative costs are allowed (or equivalently, if no positive transfers are permitted), m is defined as the largest integer for which the following inequality is satisfied:

$$\frac{1}{m-1}\left(\sum_{i=1}^{m-1} u_i - u_{soc}\right) < u_m \quad (8)$$

having assumed, without loss of generality, that players are ordered such that $u_1 \geq u_2 \geq \ldots \geq u_n$.

The Nash bargaining solution can be computed in distributed fashion.

Mesh Network Application

We consider full-mesh network topologies with 50 nodes randomly distributed on a 1000×1000 square area and 20 players (source/destination pairs).

The cost of each link is equal to its length, and the numerical results, averaged over 20 random extractions, are reported in the table.

Game	$\alpha = 0$	$\alpha = 1$	$\alpha = 10$	$\alpha = 50$	$\alpha = 100$	$\alpha = 1000$	**ILP**
SAND	6956.17	6124.62	6037.53	6043.82	6050.94	6046.66	**4214.63**
NAD-SAND	5718.86	4707.48	4214.63	4214.63	4214.63	4214.63	

Overlay Network Application

We model overlay networks over three real ISP topologies mapped using Rocketfuel.

Table: Rocketfuel-inferred ISP topologies: number of network nodes and links.

Network	Location	Nodes	Links
Telstra	AU	108	306
Sprintlink	US	141	748
Abovenet	US	315	1944

The link costs are provided by Rocketfuel, and we performed 20 random selections of 20 source/destination pairs.

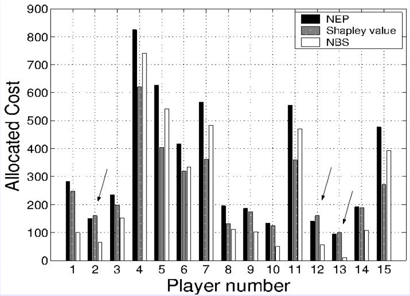
Overlay Network Application

Network	Game	$\alpha = 0$	$\alpha = 1$	$\alpha = 10$	$\alpha = 50$	$\alpha = 100$	$\alpha = 1000$	**ILP**
Telstra	SAND	165.55	164.55	163.15	163.10	163.10	163.10	**157.95**
	NAD-SAND	163.25	160.70	158.55	157.95	157.95	157.95	
Sprintlink	SAND	286.25	282.60	280.90	280.30	280.30	280.30	**255.60**
	NAD-SAND	270.30	263.35	256.85	255.65	255.60	255.60	
Abovenet	SAND	301.30	292.65	287.45	287.45	287.05	287.05	**261.55**
	NAD-SAND	282.95	272.50	262.10	261.55	261.55	261.55	

Shapley value and Nash Bargaining

Let us consider again mesh network example with 15 randomly generated players.

The total network cost is equal to 5076.0 at the NEP and to 3802.7 for the Shapley value and NBS allocations.



Thank you!

Any questions?