Games of Network Selection and Resource Allocation in Wireless Access Networks

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Introduction
Outline

- A bit of Motivation
  - Evolution of End Users
  - Evolution of Network technologies
- Network Selection Problem
  - Reference Scenario
  - Congestion Game Models
  - Numerical Results
- A Brief Sketch of the joint Network Selection and Resource Allocation Problem
  - A Bi-Level Game Model
  - Numerical Results
- Conclusion
Mobile Traffic Explosion

6.8 billions users

96% Average Penetration

Source: ITU World Telecommunication /ICT Indicators database
Note: * Estimate
Mobile Traffic Explosion

Exabytes per Month

2012: 0.9 EB
2013: 1.6 EB
2014: 2.8 EB
2015: 4.7 EB
2016: 7.4 EB
2017: 11.2 EB

66% CAGR 2012–2017

Source: Cisco VNI Mobile Forecast, 2013
User Terminals Evolution

- **Evolution of the end-user terminal**
  - Internet Clients are going Mobile (Cell Phones, PDAs), 80% of “twits” coming from mobile devices\(^1\)
  - Multi-Interface Mobile Devices

- **Increasing “mobile” bandwidth demand**
  - Mobile users expect the same connectivity experience they have in wired networks
  - New KPIs (throughput, latency)
  - From voice calls to “data” calls

Wireless Networks Evolution

- Proliferation of technologies/standards

- Things are Getting Even Worse with 4G/5G Nets
  - 10x more base stations, Many many more parameters to set, Self Organizing Cells
The Network Selection Problem

- Large number of opportunities
- Large number of “competitors”
The Resource Allocation Problem

- Resource Allocation:
  - how to assign radio resources (frequency channels, technologies, time-slots, codes) to network devices

- Driving Factors
  - Cost/Revenue
  - Capacity
  - Interference
  - Coverage

- Spectrum is a scarce resource

- Other resource limitation (e.g., energy)

- Interference/competition with other operators
The network selection game
The Network Selection Problem

- Network with $n$ users covered by $m$ networks
- Available networks may be
  - Homogeneous: same technology, different channels (e.g., multiple WiFi Access Points, multiple 3G base stations)
  - Heterogeneous: different technologies
- Which is the best choice for the users?
  - Common Selection Policies based on RSS measures neglects congestion
- Contribution:
  - Models considering network congestion
  - Models capturing the interplay and competitive dynamics among accessing users
Network Selection Game (NSG)

- Selection of the “best” available network
- Players: users
- Strategies: available networks
- Cost Function: \( c_j(i, z^i_j) \)
  - Congestion: \( z^i_j = \sum_{k \in X^i_j} \omega^i_k \)
  - Being \( X^i_j \) the set of all users that select \( i \) and interfere with \( j \)
- Players are rational and selfishly try to minimize the perceived access cost
Why different users selecting the same resources could have different interferers?

If the two access points have the same frequencies!
The cost/utility of the users depends on the actual throughput. It depends on a number of parameters and cannot be computed in closed form. And so? How can we design a game? We can provide simple models and evaluate the accuracy of these models in real-world (or simulated) applications.
User Cost Function 1 - (CDMA-like Systems)

- Non-weighted congestion game
  \[ \omega^i_j = 1, \quad z^i_j = x^i_j \]

- Cost function based on the number of interferers:
  \[ c^i_j(i, z^i_j) = x^i_j \sqrt{x^i_j} \]
User Cost Function 1 - (CDMA-like Systems)

- This cost function may well represent the case of access networks characterized by "soft" capacity degradation

- e.g., CDMA-based systems under open loop power control, where the perceived quality of a transmission depends almost exclusively on the interferers number and each transmission/user congests the shared resource evenly
Exercise: what the costs of the users?
User Cost Function 2 – Rate-Based

- Non-weighted congestion game
  \[ \omega_j^i = 1, \quad z_j^i = x_j^i \]
- \( R_j^i \): nominal rate perceived by user \( j \) from network \( i \)
- \( T_j^i = \frac{1}{R_j^i} \)
- Cost function:
  \[ c_j(i, z_j^i) = T_j^i \cdot x_j^i \]
The cost function keeps into account more information than the case of only interference.

But it does not take into account the weights of the other users.
Exercise: what is the cost of the users?
User Cost Function 3 — WiFi-like Networks

- Weighted congestion game

\[ \omega_j^i = T_j^i \]
\[ z_j^i = \sum_{h \in X_j^i} T_h^i \]

- Cost function:

\[ c_j(i, z_j^i) = T_j^i \cdot z_j^i \]

\[ x_j^i = |X_j^i| \]
The accessing users may then congest the shared resource in different ways.

In IEEE 802.11 access networks, the highest interference is caused by users with the lowest rate due to the well known performance "anomaly".

Thus, it is reasonable to assume that each user congests the resources with a specific weight that depends on the inverse of the rate the user perceives.
Exercise: what the costs of the users?
Analysis of the NSG

- **NSG Properties:**
  - Congestion Game\(^2\)
  - Asymmetric (different users may have different actions)
  - Single-choice (each user can choose one network)
  - Player-specific cost functions (different users may have different cost functions)

- **NSG can be reduced to a crowding game\(^3\):**
  - symmetric, single-choice congestion game with player specific cost functions that are monotonically increasing in the congestion level

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Characterization of the Nash Equilibria (NE):

- Non-weighted Congestion games → NE always exists in pure strategy under cost function 1 and 2

- Weighted Congestion games with separable cost functions → NE always exists in pure strategy under cost function 3

\[ c_j(i, z^i_j) = T^i_j \cdot z^i_j \]

Equilibrium vs dynamical analysis (1)

- Equilibrium analysis disregards competitive dynamics
- Usually, best response dynamics (better response) are fast and users’ mobility is slower than the convergence time
  - e.g. users join the network sequentially and the changes are local
- The activation of the users is (expected to be) asynchronous
If synchronous, no convergence could be possible

Consider the following example

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>B</td>
<td>0,0</td>
<td>1,2</td>
</tr>
</tbody>
</table>
How to assess the equilibrium quality

- We aim at assessing in practice the quality of the equilibria
- We used mathematical programming to select equilibria
  - Minimizing the social cost (PoS)
  - Maximizing the social cost (PoA)
  and the optimal social solution
Model Decision Variables and Parameters

\[ a_{jk} = \begin{cases} 1 & \text{if user } j \text{ can select access point } A_k \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in \mathcal{N} \quad k \in \mathcal{A} \]

\[ f_{ji} = \begin{cases} 1 & \text{if user } j \text{ can select frequency } f_i \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in \mathcal{N} \quad i \in \mathcal{F} \]

\[ d_{ki} = \begin{cases} 1 & \text{if } A_k \text{ transmits on frequency } f_i \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in \mathcal{A} \quad i \in \mathcal{F} \]

\[ b_{jhi} = \begin{cases} 1 & \text{if users } j \text{ and } h \text{ interfere on frequency } f_i \\ 0 & \text{otherwise} \end{cases} \quad \forall i, k \in \mathcal{A} \quad i \in \mathcal{F} \quad \forall j \in \mathcal{N} \]

\[ s_{jk} = \begin{cases} 1 & \text{if user } j \text{ chooses } A_k \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in \mathcal{N} \quad k \in \mathcal{A} \]

\[ y_{ji} = \begin{cases} 1 & \text{if user } j \text{ chooses frequency } f_i \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in \mathcal{N} \quad i \in \mathcal{F} \]
Congestion Definition

- The number of all users interfering with user $j$ set on frequency $i$ is:
  \[ x^i_j = \sum_{l \in \mathcal{N}} y_{li} b_{jli} \]

- The perceived congestion by user $j$ set to frequency $i$ is:
  \[ z^i_j = \sum_{l \in \mathcal{N}} \omega^i_l y_{li} b_{jli} \]
Socially-Optimal Network Selection

\[ \min \sum_{j \in \mathcal{N}} \sum_{i \in \mathcal{A}} y_{ji}c_j \left( i, \sum_{l \in \mathcal{N}} \omega_i^l y_{li} b_{jl} \right) \]

s.t.

\[ \sum_{k \in \mathcal{A}_j} s_{jk} = 1 \quad \forall \ j \in \mathcal{N} \] \hspace{1cm} (1)

\[ \sum_{i \in \mathcal{F}_j} y_{ji} = 1 \quad \forall \ j \in \mathcal{N} \] \hspace{1cm} (2)

\[ s_{jk} d_{ki} \leq y_{ji} \quad \forall \ j \in \mathcal{N}, i \in \mathcal{F}, k \in \mathcal{A} \] \hspace{1cm} (3)

Social Cost

Assignment Feasibility Constraints
Enforcing NE

- A NE is the solution of the following decision problem:
  - Constraints (1), (2), (3) and the new constraints

\[ f_{jk} y_{ji} c_j \left( i, \sum_{l \in \mathcal{N}} \omega^i_l y_{li} b_{jl} \right) \leq c_j \left( k, \sum_{l \in \mathcal{N}} \omega^k_l y_{lk} b_{jlk} + \omega^k_j \right) \]

\[ \forall j \in \mathcal{N}, i, k \neq i \in \mathcal{F} \quad (4) \]

If player \( j \) selects network \( i \), then the cost perceived by the user is less or equal to the cost that he would perceive changing the chosen network.
We leverage the concepts of *Price of Stability* (PoS) and *Price of Anarchy* (PoA)

\[
\text{PoS} = \frac{\text{Cost Best NE}}{\text{Optimum Cost}} \quad \text{PoA} = \frac{\text{Cost Worst NE}}{\text{Optimum Cost}}
\]

\[1 \leq \text{PoS} \leq \text{PoA}\]

The worst and best NE can be derived by maximizing/minimizing the social cost subject to constraints (1),(2),(3) and (4)
Experimental setting

Uniform Topologies
50 users (|N|)
10 networks (|A|)
L=500m
R=100m

Unbalanced Topologies
50 users (|N|)
10 networks (|A|)
L=500m
R=100m

Corridor Topologies
50 users (|N|)
5 networks (|A|)
L=600m
R=100m
Numerical evaluation on the quality of the NE shows that PoS and PoA are very close to one:

- even if users play selfishly, they reach a solution that (in terms of social utility) is very close to the optimum!

<table>
<thead>
<tr>
<th>Function</th>
<th>Uniform (a)</th>
<th>Non uniform (b)</th>
<th>Corridor (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PoS</td>
<td>PoA</td>
<td>PoS</td>
</tr>
<tr>
<td>1</td>
<td>1.00000</td>
<td>1.00733</td>
<td>1.00000</td>
</tr>
<tr>
<td>2</td>
<td>1.00497</td>
<td>1.01079</td>
<td>1.00493</td>
</tr>
<tr>
<td>3</td>
<td>1.00000</td>
<td>1.00563</td>
<td>1.00020</td>
</tr>
</tbody>
</table>
A note

- We did not derive upper bounds for PoA and PoS
- For cost function 1, an instance with PoA arbitrarily close to 2 can be found
- A simple example in which PoA = 5/3 follows
  - In practice, PoA and PoS are never close to the bound when a reasonable number of users is present
  - The example motivates the study of the corridor topology
Exercise: is PoA > 1?
Exercise: different dynamics, different equilibria
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Exercise: is PoS $> 1$?

- Prove that PoS for cost function 2 is $> 1$
- Prove that PoS for cost function 3 is $> 1$
Experimental results by simulation

- We used NS2 to simulate the network traffic and evaluate the actual costs of the user.
- The aim is to access whether the models are accurate and the performance of the protocols.
Actual throughput perceived by users

Asymmetric case
50 users
2 networks
L = 500
R = 100
Actual throughput perceived by users

50 users
10 networks
L = 500
R = 100

UDP Load per user [Mbit/s]

Throughput per user [Mbit/s]

Uniform BE

Cost Function 1
Cost Function 2
Cost Function 3
Nearest AP
Actual throughput perceived by users

Corridor
50 users
10 networks
L = 500
R = 100
Jain’s Fairness index

\[ f = \frac{\left(\sum_{j \in \mathcal{N}} \theta_j \right)^2}{n \cdot \sum_{j \in \mathcal{N}} \theta_j^2} \]

Uniform BE
Best response dynamics length

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    xlabel=Number of users,
    ylabel=Dynamic length,
    legend entries={m=3, m=6, m=9, m=12, m=15},
]
\addplot coordinates {
    (0,0) (1,0) (2,0) (3,0) (4,0) (5,0) (6,0) (7,0) (8,0) (9,0) (10,0) (11,0) (12,0) (13,0) (14,0) (15,0) (16,0) (17,0) (18,0) (19,0) (20,0) (21,0) (22,0) (23,0) (24,0) (25,0) (26,0) (27,0) (28,0) (29,0) (30,0) (31,0) (32,0) (33,0) (34,0) (35,0) (36,0) (37,0) (38,0) (39,0) (40,0) (41,0) (42,0) (43,0) (44,0) (45,0) (46,0) (47,0) (48,0) (49,0) (50,0) (51,0) (52,0) (53,0) (54,0) (55,0) (56,0) (57,0) (58,0) (59,0) (60,0) (61,0) (62,0) (63,0) (64,0) (65,0) (66,0) (67,0) (68,0) (69,0) (70,0) (71,0) (72,0) (73,0) (74,0) (75,0) (76,0) (77,0) (78,0) (79,0) (80,0) (81,0) (82,0) (83,0) (84,0) (85,0) (86,0) (87,0) (88,0) (89,0) (90,0) (91,0) (92,0) (93,0) (94,0) (95,0) (96,0) (97,0) (98,0) (99,0) (100,0) (101,0) (102,0) (103,0) (104,0) (105,0) (106,0) (107,0) (108,0) (109,0) (110,0) (111,0) (112,0) (113,0) (114,0) (115,0) (116,0) (117,0) (118,0) (119,0) (120,0) (121,0) (122,0) (123,0) (124,0) (125,0) (126,0) (127,0) (128,0) (129,0) (130,0) (131,0) (132,0) (133,0) (134,0) (135,0) (136,0) (137,0) (138,0) (139,0) (140,0) (141,0) (142,0) (143,0) (144,0) (145,0) (146,0) (147,0) (148,0) (149,0) (150,0) (151,0) (152,0) (153,0) (154,0) (155,0) (156,0) (157,0) (158,0) (159,0) (160,0) (161,0) (162,0) (163,0) (164,0) (165,0) (166,0) (167,0) (168,0) (169,0) (170,0) (171,0) (172,0) (173,0) (174,0) (175,0) (176,0) (177,0) (178,0) (179,0) (180,0) (181,0) (182,0) (183,0) (184,0) (185,0) (186,0) (187,0) (188,0) (189,0) (190,0) (191,0) (192,0) (193,0) (194,0) (195,0) (196,0) (197,0) (198,0) (199,0) (200,0)
};
\end{axis}
\end{tikzpicture}
\end{center}
The resource allocation game
Networks take part to the game

- Resource allocation (frequency) is not fixed
- Access networks are brought into the competition:
  - **Players**: users/networks
  - **Strategies**: available networks/available frequencies
  - **Cost/Utility Function**: same presented before (users) and number of connected users (networks)

- Game is a **two-stage game** (networks play first, then come users)
- We are interested in finding a **subgame perfect Nash Equilibrium**
Example: cost function 1

Different frequencies
Example: cost function 1

Different frequencies

Same frequencies
Congestion = 2
Congestion = 2
Example: cost function 1

Different frequencies

Same frequencies

Congestion = 2

Congestion = 2
Example: cost function 1

Different frequencies

Congestion = 1

Same frequencies

Congestion = 2
Example: cost function 1

Same frequencies

Congestion = 2

Congestion = 2

Congestion = 2
The network game with cost function 1

Multiple equilibria in the user game!

<table>
<thead>
<tr>
<th></th>
<th>f1</th>
<th>f2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1</td>
<td>?</td>
<td>1,1</td>
</tr>
<tr>
<td>f2</td>
<td>1,1</td>
<td>?</td>
</tr>
</tbody>
</table>
The subgame perfect equilibrium

Users NE reached fixing the networks strategies.
Pure strategy equilibrium existence

- Cost Function 1: the equilibrium always exists
- Cost Function 2 and 3: the equilibrium may NOT exist
Pure strategy equilibrium existence

- Cost Function 1: the equilibrium always exists
- Cost Function 2 and 3: the equilibrium may NOT exist

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<tbody>
<tr>
<td>f1</td>
<td>2,0</td>
<td>1,1</td>
</tr>
<tr>
<td>f2</td>
<td>1,1</td>
<td>2,0</td>
</tr>
</tbody>
</table>
Pure strategy equilibrium existence

- **Cost Function 1:** the equilibrium always exists
- **Cost Function 2 and 3:** the equilibrium may NOT exist

- A and B use the same frequency
  - A gets 2 users and B none
- A and B use different frequencies
  - A and B get 1 user

\[ T_1^A < T_1^B < 2T_1^A \]

\[ c_u(A) = 2T_1^A < c_u(B) = 2T_1^B \]

\[ c_u(A) = 2T_1^A > c_u(B) = T_1^B \]
The $\varepsilon$-equilibrium

- Network game in normal form

<table>
<thead>
<tr>
<th>A\B</th>
<th>F1</th>
<th>F2</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>(2,0)</td>
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</tr>
<tr>
<td>F2</td>
<td>(1,1)</td>
<td>(2,0)</td>
</tr>
</tbody>
</table>

In order to guarantee the existence of a stable state, we introduce the $\varepsilon$-equilibrium with $\varepsilon=1$, i.e., a network deviates only if it can improve its payoff of at least 2 users!

Equilibrium does NOT exist !!
The \( \varepsilon \)-equilibrium (cont’d)

- We introduce the concept of \( \varepsilon \)-equilibrium (i.e., it is not possible for any player to gain more than \( \varepsilon \) by unilaterally deviating from his strategy) on the network equilibrium constraint.

- Using an extension of the mathematical formulation previously introduced, we characterize the probability of non-existence and the minimum value of \( \varepsilon \).
Probability of non-existence

- 2 networks
- 2 available frequencies
- Cost Function 3
Normalized $\varepsilon$

Number of users that a network “loses”
2 Networks
2 frequencies
Cost Function 3
Competition cost

![Graph showing the average cost per user with different scenarios: Equilibrium with CF 2, Equilibrium with CF 3, Same Frequencies, Different Frequencies.]
Conclusions
Conclusion and Follow Ups

- **Congestion game models** have been studied for the network selection and resource allocation games.
  - Analytical and numerical results have been derived in order to assess the quality of the NE.

- **Future/ongoing work**
  - Network Selection in **Wireless Mesh Networks**
  - Leverage game theory for protocol design
  - Work in the field of **Cognitive Radio Networks**
  - What about **Infrastructure Sharing**?
